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INSTYTUT MASZYN PRZEPŁYWOWYCH

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

102

GDAŃSK 1997

THE TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

exist for the publication of theoretical and experimental investigations of all aspects of the mechanics and thermodynamics of fluid-flow with special reference to fluid-flow machines

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PRACE INSTYTUTU MASZYN PRZEPŁYWOWYCH

poświęcone są publikacjom naukowym z zakresu teorii i badań doświadczalnych w dziedzinie mechaniki i termodynamiki przepływów, ze szczególnym uwzględnieniem problematyki maszyn przepływowych

Wydanie publikacji dofinansowane zostało przez PAN ze środków DOT uzyskanych z Komitetu Badań Naukowych

EDITORIAL OFFICE – REDAKCJA

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ISSN 0079-3205

TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

1997 No. 102

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Impingement of two-phase gas-liquid jet on a flat plate[2]

A theoretical analysis has been conducted of motion of a laminar liquid film induced by the impingement of a gaseous jet with droplets on a flat surface. Splitting of the Newtonian liquid on the surface is caused by the motion of gas a flowing under a pressure gradient over the liquid film. A theoretical model of the phenomenon and a method of solving have been postulated. The obtained results have been compared with the experimental data of other authors.

Nomenclature

a	–	coefficient, equation 2.5,	p	–	pressure,
C_f	–	friction coefficient,	u	–	circumferential velocity,
$C(r)$	–	coefficient, equation 2.20,	v	–	radial velocity,
D	–	nozzle diameter,	w	–	velocity in direction z,
g	–	acceleration of gravity,	r, ϕ, z	–	cylindrical co-ordinates,
H	–	height of nozzle suspension over the surface,	t	–	time,
			δ	–	liquid layer thickness,
Q	–	volumetric flow rate of the liquid,	μ	–	dynamic viscosity,
			ρ	–	liquid density,
q	–	unit volumetric flow rate of the liquid,	τ	–	wall shear stress,
			$Re_s = \frac{\nu_s \delta_s \rho}{\mu}$		Reynolds number.

Subscripts

i	–	gas-liquid surface,	o	–	ambient, characteristic quantity,
g	–	gas,	w	–	wall,
φ	–	circumferential direction,	s	–	stabilisation, circumferential direction,
r	–	radial direction,	$+$	–	non-dimensional quantity.
z	–	perpendicular direction to the surface,			

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[2]The work was sponsored by a research project KBN 3 P404 034 07

1. Introduction

Surface wetting by impinging two-phase liquid-gas jet is widely used for intensive cooling. The cooling process is particularly intensive when the phenomenon of film evaporation takes place. Such surface cooling can be found in heat exchangers, chemical engineering apparatuses or electronic devices exposed to large loads in high power computers. Splitting of the liquid formed from the droplets on the wall has been scarcely investigated both experimentally and theoretically. Up to date it has not found a complete explanation. There is no precise mathematical model of this phenomenon. Works known from the literature [1-2] give very simplified models of the phenomenon. The interia forces are neglected. However, this assumption is not justified as approaching the wall, the two-phase jet reduces its kinetic energy at the same time increasing the pressure. At the point of impingement the kinetic energy is equal to zero and the pressure has a maximum value. From the stagnation point there is a merely radial gas flow. The pressure decreases and the radial velocity of the gas increases. There is, therefore, a pressure gradient in the radial direction. As a result of increased inertia the atomised liquid phase suspended in the gas in the form of droplets deviates with respect to gas streamlines and separates onto the wall. The droplets form a liquid film which splits radially. In the vicinity of the stagnation point the flow is laminar. In the work, laminar liquid films on the wall have been considered. The process of splitting of the liquid impinging on a horizontal surface with a specified velocity has been investigated theoretically and experimentally in [4,5]. The aim of the present work is to investigate splitting of the liquid formed from the two-phase jet impingement on a surface. The developed model draws on integral equations of motion of the liquid film near the wall. Numerical calculations have also been performed to illustrate the method.

2. Analysis

A flat surface impinged by a two-phase jet is presented together with a co-ordinate system in Fig. 1. A cylindrical system of co-ordinates r, z has been assumed. The co-ordinate z is perpendicular to the surface. The flow of the liquid and gas jet flowing out of a nozzle with the rate Q is considered two dimensional, axisymetrical.

It has been assumed that:

1. The motion of the liquid film is laminar, fully developed and induced by the motion of the gaseous phase.
2. The gaseous phase flowing out axisymmetrically from the nozzle induces the shear stress on the liquid film surface and feeds the film with the liquid at the rate q.
3. Liquid-wall and liquid-gas separation surfaces are smooth.

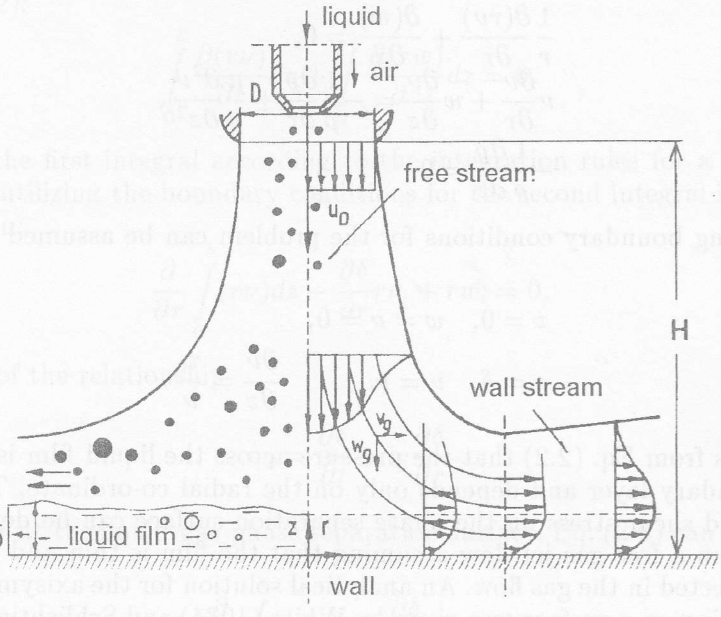

Fig. 1. Impingement of two-phase gas-liquid jet.

4. Physical properties of the liquid are constant.
5. The liquid film thickness is small, which allows us to assume a linear velocity profile in z direction and simplify the equation of motion as in the case of the boundary layer.
6. Gravity effects are neglected.
7. The circumferential velocity component is zero as a result of symmetry ($u = 0$). It means that the radial component and film thickness are functions of the radius r.

A full set of conservation equations of mass and momentum in cylindrical co-ordinates is given in the following way [5]:

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial(\rho r \nu)}{\partial r}+\frac{\partial(\rho u)}{\partial \phi}+\frac{\partial(\rho w)}{\partial z}=0$$

$$\frac{\partial u}{\partial t}+\nu\frac{\partial u}{\partial r}+\frac{u}{r}\frac{\partial u}{\partial \phi}+\frac{\nu u}{r}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho r}\frac{\partial p}{\partial \phi}+\frac{\mu}{\rho}\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(ru)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 u}{\partial \phi^2}+\frac{2}{r^2}\frac{\partial \nu}{\partial \phi}+\frac{\partial^2 u}{\partial z^2}\right\}+g_\phi$$

$$\frac{\partial \nu}{\partial t}+\frac{u\partial \nu}{r\partial \phi}-\frac{u^2}{r}+\nu\frac{\partial \nu}{\partial r}+w\frac{\partial \nu}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{\rho}\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\nu)\right]+\frac{1}{r^2}\frac{\partial^2 \nu}{\partial \phi^2}-\frac{2}{r^2}\frac{\partial u}{\partial \phi}+\frac{\partial^2 \nu}{\partial z^2}\right\}+g_r$$

$$\frac{\partial w}{\partial t}+\nu\frac{\partial w}{\partial r}+\frac{u}{r}\frac{\partial w}{\partial \phi}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 w}{\partial \phi^2}+\frac{\partial^2 w}{\partial z^2}\right\}+g_z. \qquad (2.1)$$

According to the above assumptions, the governing equations can be simplified as below

$$\frac{1}{r}\frac{\partial(r\nu)}{\partial r}+\frac{\partial(w)}{\partial z}=0,$$
$$\nu\frac{\partial\nu}{\partial r}+w\frac{\partial\nu}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{\rho}\frac{\partial^2\nu}{\partial z^2}, \tag{2.2}$$
$$-\frac{1}{\rho}\frac{\partial p}{\partial z}=0.$$

The following boundary conditions for the problem can be assumed

$$z=0,\quad w=\nu=0,$$
$$z=\delta,\quad \nu=\nu_i,\quad \frac{\partial\nu}{\partial z}=\frac{\tau_i}{\nu} \tag{2.3}$$

It comes from Eq. (2.2) that the pressure across the liquid film is constant as in the boundary layer and depends only on the radial co-ordinate. The pressure gradient and shear stress on the phase separation surface can be determined for the film from a free gas jet flow assuming that the film is thin and its thickness can be neglected in the gas flow. An analytical solution for the axisymmetrical gas flow impinging on a surface was given by White (1974) and Schlichting (1980) [1]. It was developed as the solution of a boundary layer flow with a pressure gradient. The pressure distribution along the wall can be written after Schlichting as

$$p_g=p_0-\frac{1}{2}\rho_g a^2(r^2+f(z)). \tag{2.4}$$

The velocity components for the gas flow are given by the relations, see Martin (1977), [1]:

$$\nu_g=arF'(\eta),\quad w_g=-2az,$$
$$a\approx\left(\frac{V_D}{D}\right)\left(1.04-0.034\frac{H}{D}\right),\quad \eta=z\sqrt{\frac{a\rho_g}{\mu_g}} \tag{2.5}$$

Martin suggested that for an axisymmetrical nozzle the value of the coefficient can be expressed as a function of geometrical parameters as in Fig. 1.
The shear stress on the gas-liquid phase separation surface can be calculated based on the gas flow and is equal to

$$\tau_i=\mu_g\frac{\partial\nu_g}{\partial z}\bigg|_0=1.312\sqrt{a^3\rho_g\mu_g r} \tag{2.6}$$

because $F''(0)=1.312$.

In order to find an approximate solution of the problem, integral forms of conservation equations have been developed [3].

The mass balance within the liquid film can be obtained by integrating the first equation (2.2):

$$\int\limits_0^\delta \frac{\partial(r\nu)}{\partial r}dz + \int\limits_0^\delta \frac{\partial(rw)}{\partial z}dz = 0. \tag{2.7}$$

Integrating the first integral according to the integration rules for a parametric integral and utilizing the boundary conditions for the second integral in Eq. (2.7) we can obtain:

$$\frac{\partial}{\partial r}\int\limits_0^\delta (r\nu)dz - \frac{\partial \delta}{\partial r} r\nu_i + rw_i = 0. \tag{2.8}$$

Making use of the relationship

$$w_i = \frac{\partial \delta}{\partial t} + \nu_i \frac{\partial \delta}{\partial r} \tag{2.9}$$

being in force at the liquid-gas phase separation surface, Eq. (2.8) can be brought to the form

$$\frac{\partial}{\partial r}\int\limits_0^\delta (r\nu)dz + \frac{\partial \delta}{\partial t} r = 0. \tag{2.10}$$

Assuming that

$$\frac{\partial \delta}{\partial t} = -q = const. \tag{2.11}$$

and placing Eq. (2.11) into (2.10), we can obtain the following mass conservation equation

$$\frac{\partial}{\partial r}\left[r\int\limits_0^\delta \nu dz\right] - qr = 0. \tag{2.12}$$

Integrating Eq. (2.12) from 0 to r and multiplying by 2π we can get the mass equation as given in [1]

$$2\pi r \int\limits_0^\delta \nu dz = \pi r^2 q. \tag{2.13}$$

The integral momentum equation can be obtained by integrating the components of the second equation (2.2) within the film thickness limits.
Integrating and rearranging the first component gives

$$\int\limits_0^\delta \nu \frac{\partial \nu}{\partial r} dz = \frac{1}{2}\int\limits_0^\delta \frac{\partial(\nu^2)}{\partial r}dz = \frac{1}{2}\left[\frac{\partial}{\partial r}\int\limits_0^\delta \nu^2 dz - \frac{\partial \delta}{\partial r}\nu_i^2\right]. \tag{2.14}$$

Integrating by parts the second component and considering the differential mass balance equation, we can obtain

$$\int_0^\delta w\frac{\partial \nu}{\partial r}dz = w_i\nu_i + \int_0^\delta\left(\nu\frac{\partial \nu}{\partial r}+\frac{\nu^2}{r}\right)dz = w_i\nu_i + \frac{1}{2}\left[\frac{\partial}{\partial r}\int_0^\delta \nu^2 dz - \frac{\partial \delta}{\partial r}\nu_i^2\right] + \frac{1}{r}\int_0^\delta \nu^2 dz. \tag{2.15}$$

The third and fourth components after the integration assume the following forms

$$\int_0^\delta \frac{\mu}{\rho}\frac{\partial^2 \nu}{\partial z^2}dz = \frac{\mu}{\rho}\frac{\partial \nu}{\partial z}\bigg|_0^\delta = \frac{\tau_i}{\rho} - \frac{\tau_w}{\rho}, \tag{2.16}$$

$$\int_0^\delta \frac{1}{\rho}\frac{\partial p}{\partial r}dz = \frac{1}{\rho}\frac{\partial p}{\partial r}\delta. \tag{2.17}$$

Summing all transformed components of the equation of motion we can obtain

$$\frac{\partial}{\partial r}\int_0^\delta \nu^2 dz - \nu_i^2\frac{\partial \delta}{\partial r} + w_i\nu_i + \frac{1}{r}\int_0^\delta \nu^2 dz = \frac{\tau_i - \tau_w}{\rho} - \frac{1}{\rho}\frac{\partial p}{\partial r}\delta. \tag{2.18}$$

Introducing the expressions (2.9) and (2.11) into (2.18) gives

$$\frac{\partial}{\partial r}\int_0^\delta \nu^2 dz - \nu_i q + \frac{1}{r}\int_0^\delta \nu^2 dz = \frac{\tau_i - \tau_w}{\rho} - \frac{1}{\rho}\frac{\partial p}{\partial r}\delta. \tag{2.19}$$

In order to solve the problem let us assume a simplest, linear velocity profile in the form:

$$\nu = C(r)z. \tag{2.20}$$

The coefficient $C(r)$ and the film thickness are determined from the integral balance equations (2.13) and (2.19). Placing Eq. (2.20) into (2.13), we can obtain

$$C(r) = \frac{qr}{\delta(r)^2}, \tag{2.21}$$

and also

$$\tau_w = \mu\frac{\partial \nu}{\partial z} = C(r)\mu = \frac{qr\mu}{\delta(r)^2}. \tag{2.22}$$

Let us assume that the inertia forces are small and can be neglected (left-hand side of Eq. (2.19)). Then combining Eqs.(2.6) and (2.20) with Eqs.(2.21) and

(2.22) and the formulaa for pressure gradient (2.4), we can finally arrive at an expression

$$q = 1.312\sqrt{\frac{a^3\rho_g\mu_g}{\mu^2}\delta^2 + \frac{a^2\rho_g}{\mu}\delta^3}, \tag{2.23}$$

analogical to the one obtained in [1] with the only difference that Eq. (2.23) does not exhibit a coefficient 2/3 in the second component on the right-hand side which appears in [1]. Substituting the expressions (2.6), (2.20) into the full equation of motion (2.19), we can obtain

$$\frac{d\delta}{dr} = \frac{3}{q^2 r}\left(\frac{1.312\sqrt{a^3\rho_g\mu_g r_0 \delta^2}}{\rho} + \frac{\rho_g a^2\delta^3}{\rho} - \frac{q\mu}{\rho}\right). \tag{2.24}$$

Introducing non-dimensional variables

$$r^+ = \frac{r}{r_0} \qquad \delta^+ = \frac{\delta}{r_0}, \tag{2.25}$$

where: r_0 is the characteristic radius, the expressions (2.23) and (2.24) can be transformed as follows

$$q = 1.312 r_0^2\sqrt{\frac{a^3\rho_g\mu_g}{\mu^2}\delta^{+2} + \frac{a^2\rho_g}{\mu}r_0\delta^{+3}}, \tag{2.26}$$

$$\frac{d\delta^+}{dr^+} = -\frac{3}{q^2 r^+}\left(\frac{1.312\sqrt{a^3\rho_g\mu_g r_0\delta^2}}{\rho} + \frac{\rho_g a^2 r_0^2\delta^{+3}}{\rho} - \frac{q\mu}{\rho}\right). \tag{2.27}$$

The film thickness after its stabilisation is determined by the relation (2.26). In order to solve Eq. (2.27), it is necessary to know the film thickness for a zero radius or the film thickness along the development length, when the inertia forces can be neglected. As the experiments show, the film thickness in the jet centre is not zero.

The wall shear stress can be expressed in general as:

$$\tau_w = C_f\rho\frac{\nu}{2}. \tag{2.28}$$

Then, if the friction coefficient is known, we can use Eq. (2.19) in the laminar and turbulent flow.

The expression (2.19) together with the mass balance equation (2.13) form a closed set of equations which enables the determination of the local film thickness.

Integrating Eq. (2.27), we can obtain the dependence between the liquid layer thickness and radius. The direction of the integral curve is determined by the derivative of the liquid film thickness with respect to the radius. This derivative tends to infinity when the denominator in Eq. (2.27) reaches zero.

When the nominator and denominator in the expression (2.27) simultaneously go to zero, the derivative which determines the local direction is undefined and a singular point appears. In this case, the behaviour of integral curves in the vicinity of a singular point can be investigated by linearisation of the right-hand side of Eq. (2.27) [4].

3. Numerical calculations

The numerical calculations have been performed for the parameters corresponding to the available experimental data [1], i.e. for the jet of water droplets atomized in the flow of air through a nozzle. The value of the parameter a in Eq. (2.5) has been estimated at $a = 1500\mathrm{s}^{-1}$. It has been assumed that the atmospheric pressure is equal to $p_g = 240$ kPa and the characteristic radius is $r_0 = 2.10^{-4}$m.

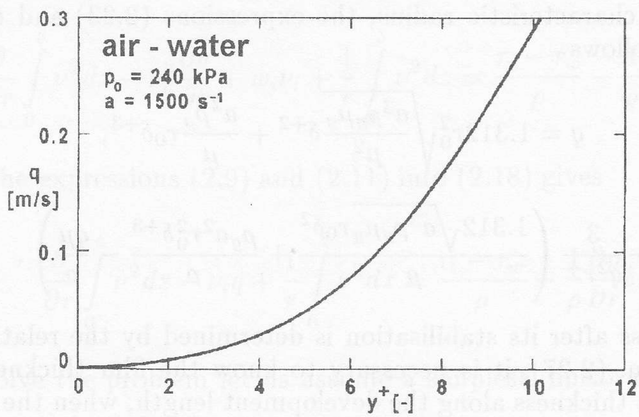

Fig. 2. Unit wetting volume q vs. non-dimensional thickness y^+ of the stabilised liquid film.

For the above data, the thickness of a fully developed liquid film as a function of wetting rate q has been determined with the help of Eq. (2.26). The results are given in Table 1 and Fig. 2. From there, knowing the wetting rate, we can determine the thickness of the fully developed liquid film. If, for example, we assume that the unit volumetric flow rate of water droplets is $q = 1$ litre/h/cm^2=1/360 m/s, we obtain that the film thickness is $\delta_s = 200 \cdot 10^{-6}$ m. The thickness of the liquid film before stabilisation for this unit volumetric flow rate has been evaluated from Eq. (2.27). The calculations have been performed using a Runge-Kutta fourth-order method under Mathcad 5+ on PC. The results of calculations are presented in Fig. 3.

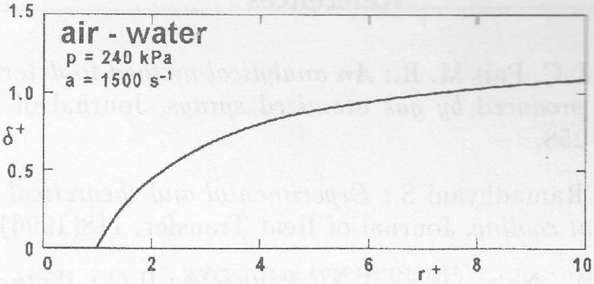

Fig. 3. Non-dimensional film thickness δ^+ vs. non-dimensional co-ordinate r^+.

Table 1. The unit wetting volume q as a function of non-dimensional thickness of the stabilised liquid film for $p = 240$ kPa, $a = 1500$ s^{-1}.

[m/s]	δ^+
0	0
0.1	$1.17576 \cdot 10^{-5}$
0.2	$4.77218 \cdot 10^{-5}$
0.5	$3.11221 \cdot 10^{-4}$
1.0	0.00133
2.0	0.00602
3.0	0.01509
4.0	0.02959
5.0	0.04815

4. Conclusions

A simple model of two-phase liquid-gas jet impinging on a flat surface and forming a thin liquid has been formulated. The model draws on the conservation equations of mass and momentum and closure equations. The model can be further developed to investigate the heat transfer at the heated surface impinged by the jet and to calculate the heat transfer coefficient to this surface. As mentioned in the introduction, this case has several practical applications, for example in heat exchangers.

The obtained equation (2.27) for the film thickness is strongly nonlinear and in some particular cases can possess singular points. In other cases the right-hand side of Eq. (2.27) contains zeros and the solution has non-monotonic regions. Therefore, before starting the numerical calculations it is required to conduct a qualitative analysis of solution in order to find the adequate method of solving the problem.

Manuscript received in August 1996

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Struga dwufazowa gazu i cieczy uderzającej o powierzchnię płaską

Streszczenie

Przeprowadzono analizę teoretyczną laminarnego ruchu filmu cieczowego powstałego wskutek uderzenia strugi gazu zawierającej kropelki cieczy o powierzchnię płaską. Rozpływ filmu cieczy newtonowskiej na powierzchni powodowany jest ruchem gazu przepływającego z gradientem ciśnienia nad filmem cieczowym. Sformułowano model teoretyczny zjawiska, wyprowadzono całkowe równania zachowania dla tego przypadku oraz zaproponowano metodę rozwiązania zagadnienia. Wskazano na istnienie w rozwiązaniu niemonotoniczności filmu w pobliżu punktu spiętrzenia gazu. Wykonano obliczenia numeryczne ilustrujące teorię. Uzyskany rezultat dla grubości filmu ustabilizowanego porównano z istniejącą teorią zagadnienia [1] uzyskując dobrą zgodność wyników.