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TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

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3D inviscid flutter of IV Standard Configuration. Part II. Coupled fluid-structure oscillations

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Abstract

A three-dimensional nonlinear time-marching method and numerical analysis for aeroelastic behaviour of oscillating blade of the IV Standard Configuration has been presented. The approach is based on the solution of the coupled fluid-structure problem in which the aerodynamic and structural equations are integrated simultaneously in time. In this formulation of a coupled problem, the interblade phase angle at which stability (or instability) would occur, is a part of the solution. The ideal gas flow through multiple interblade passage (with periodicity on the whole annulus) is described by the unsteady Euler equations in the form of conservative laws, which are integrated by use of the explicit monotonous second order accurate Godunov-Kolgan finite volume scheme and a moving hybrid H-H (or H-O) grid. The structure analysis uses the modal approach and 3D finite element model of the blade. The blade motion is assumed to be a linear combination of mode shapes with the modal coefficients depending on time. The influence of the natural frequencies on the aeroelastic coupled oscillations for the Fourth Standard Configuration is shown. It has been shown that interaction between modes plays an important role in the aeroelastic blade response. This interaction has essentially nonlinear character and leads to blade limit cycle oscillations. The sign of the aerodamping coefficient calculated for the harmonic oscillations, may be considered only as a necessary, but not a sufficient condition for self-excited oscillations.

Keywords: Flutter; Blades; Inviscid flow

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1 Introduction

Aeroelasticity phenomena are characterised by the interaction of fluid and structural domains. In recent times, the new approaches based on simultaneous integration in time of the equations of motion for the structure and the fluid have been developed (Bakhle et al. [1], Bendiksen [2, 3], Carstens and Belz [5], He [6, 7], Kielb [10], Gnesin [9], Rzdkowski et al. [13], Vahdati et al. [15]. These approaches are very attractive due to the general formulation of a coupled problem, as the interblade phase angle at which stability (instability) would occur is a part of solution.

In the present study the simultaneous time integration method has been described to calculate the aeroelastic behaviour for a three-dimensional oscillating blade row of the Fourth Standard Configuration in the transonic gas flow.

All calculations were run from the very beginning for harmonic oscillations with the assumed natural frequencies corresponding to the assumed mode shapes and the interblade phase angle as the initial conditions. After some time lapse, named as the start regime, there began the coupled oscillations, where the blade displacements and velocities, and the flow parameters are used as initial conditions for the coupled time integration procedure. The interblade phase angle and frequency of vibration are part of the solution of the problem.

The influence of natural frequencies on the aeroelastic coupled oscillations is presented. The sign of the aerodamping coefficient calculated for the harmonic oscillations may be considered only as a necessary but not sufficient condition for self-excited oscillations. It has been shown that interaction between modes plays an important role in the aeroelastic blade response. This interaction has essentially a nonlinear character and leads to blade limit cycle oscillations and to changing of the interblade phase angle.

2 Aerodynamic model

The 3D transonic flow of an ideal gas through a space multipassage blade row is considered. In a general case the flow is assumed to be aperiodic function from blade to blade (in pitchwise direction), so the calculated domain includes all blades of the whole assembly.

The ideal gas flow around blade row is described by the unsteady 3D Euler equations in conservative form, which are integrated using the explicit monotonous second-order accurate Godunov-Kolgan, finite volume scheme and moving grids.

The aerodynamic model was presented in details by Gnesin and Rzdkowski [8].

3 Structural model

The structural model is based on a linear model, the mode shapes and natural frequencies being obtained via standard FE analysis techniques. The mode shapes are interpolated from the structure mesh onto the aerodynamic mesh.

The structural part of the aerodynamic equations of motion is uncoupled by using the mode shape matrix and the modal superposition method (see Rządkowski [12], Gnesin and Rządkowski [8]).

Boundary conditions from the structural and aerodynamic domains are exchanged at each time step and the aerodynamic mesh is moved to follow the structure motion (the partially coupled method). The structural damping is not included here. The scheme used to integrate the structural equations is the same as the scheme used in the flow code. For this scheme the accuracy of the calculations of natural frequencies and mode shapes is sufficient. The integration scheme introduces damping; this value is very small and was found from calculations done with the aerodynamic forces set to zero.

4 Numerical results

Numerical calculations have been carried out for the turbine cascade known as the Fourth Standard Configuration, which has been experimentally investigated in the nonrotating annular cascade tunnel in transonic flow (Bölcs and Fransson [4]). The numerical and experimental verification of the numerical code presented here can be found in [14].

In the harmonic calculation (see [16]) it was found that the areodamping coefficient is negative (the flutter condition) in the area close to the interblade phase angle equal to −90 deg for the 1st mode and to +90 deg for the second mode. The vibration of the cascade according to the third and fourth modes shapes with different natural frequencies and the interblade phase angles is stable.

The considered blade has natural frequencies equal to 150, 750, 900, 1050 Hz respectively.

All calculations were run from the very beginning for harmonic oscillations, until the steady state periodic flow through vibrating blade row is converged. During this time the forced frequencies of harmonic oscillations of each mode were equal to their natural frequencies respectively and the blades are vibrating with the assumed interblade phase angles. After some time lapse, named as the start regime, there began the coupled vibrations in which the blade displacements and velocities, and the flow parameters are used as initial conditions for the coupled time-integration procedure. The interblade phase angle and frequency of vibration are part of the solution of the problem.

The purpose of the investigations presented here is to study the influence of the first four natural modes on each other and on the blade response for the assumed initial interblade phase angles equal to +90 deg and −90 deg, as the initial condition in the harmonic oscillations.

Let us consider the aeroelastic blade response, vibrating from the beginning at the harmonic oscillations with IBPA of −90 deg, and next according to coupled fluid-structure interaction.

The aeroelastic response of the two adjacent blades vibrating according to the 1st mode, the excitation frequency of 150 Hz and IBPA equal to −90 deg is presented in Fig. 1. These parameters correspond to aerodamping coefficients equal to zero, see [14]. The harmonic oscillation continues through one period then the coupled vibrations begin. Here q_{11} and q_{21} denote the modal coefficients for the first and second blade oscillating in the 1st mode. As it can be seen from Fig. 1 the blade oscillations are damped with the logarithmic decrement equal to approximately a constant value. It indicates that the blade motion is close to the linear damped oscillations. The blades are fixed at the root, so the root conditions are different from the experiment presented by Bölcs and Fransson [4]. This is a reason for the damped oscillations instead of typical flutter response.

The work coefficient for each of blades (four passages in this case) is shown in Fig. 2 and for the blade row as a whole is presented in Fig. 3. The monotonous convergence of the work coefficient to zero demonstrates the dissipation of the vibrating blades energy to the flow field. The aeroelastic responses of the two adjacent blade vibrating according to the 2nd , 3rd and 4th modes, with the excitation frequencies 750, 900, 1050 Hz, respectively and IBPA equal to −90 deg are presented in Figs. 4-7. The harmonic oscillation continue through one period then the coupled vibration begin.

The work coefficient for each of blades (four passages in this case) is shown in Fig. 5. The monotonous convergence of the work coefficient to zero demonstrates the dissipation of the vibrating blades energy to the flow field. For all regimes the oscillations are damped. The higher the natural frequency the more stable the blades are.

Figure 8 shows the typical phase trajectory (in coordinate system *displacement-velocity*) for the damped oscillations of the cascade according to the third mode shape. The bold curve corresponds to harmonic oscillations, the thin one is the spiral convergent to the stable state placed in the origin of coordinates.

The different character of blade motion was observed for consideration of the interaction of the natural mode vibrations. Figure 9 shows the blade response at oscillations corresponding to the first and second modes together. It is clearly observed that response of the blade for the 2nd mode shape decays, but the amplitude of the 1st mode tends to the approximately constant value, that corresponds

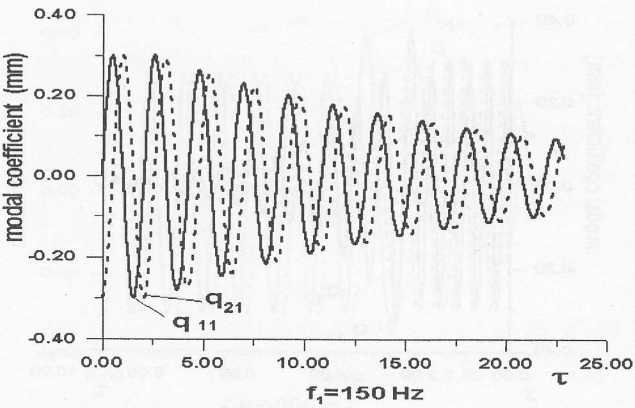

Figure 1. The aeroelastic oscillations of two adjacent blades for the first mode, and IBPA −90 deg as the initial condition in the harmonic motion, $t = \tau/316$ s.

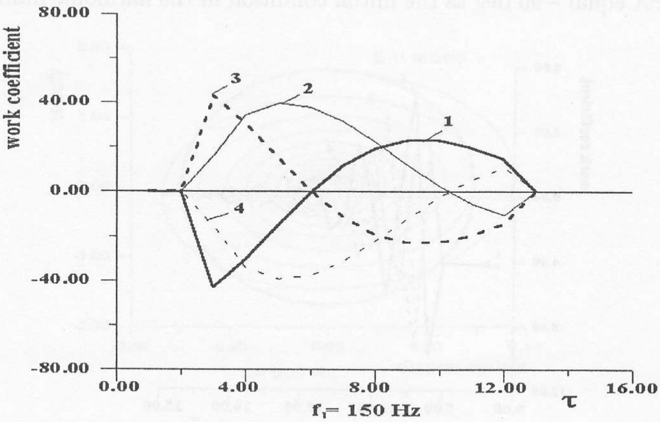

Figure 2. The work coefficient for all vibrating blade (1st mode), and IBPA −90 deg as the initial condition in the harmonic motion, $t = \tau/316$ s.

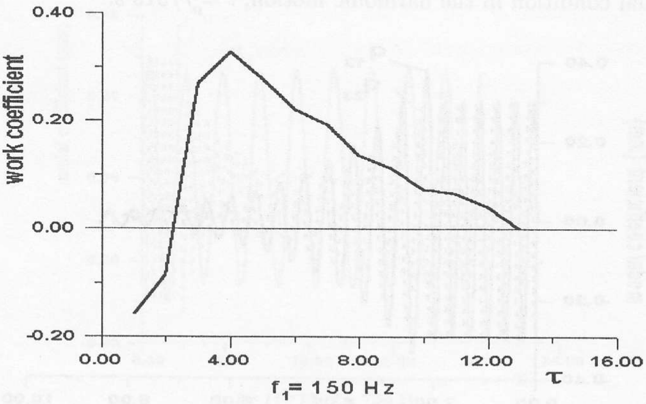

Figure 3. The total work coefficient for the blade row (1st mode) and IBPA −90 deg as the initial condition in the harmonic motion, $t = \tau/316$ s.

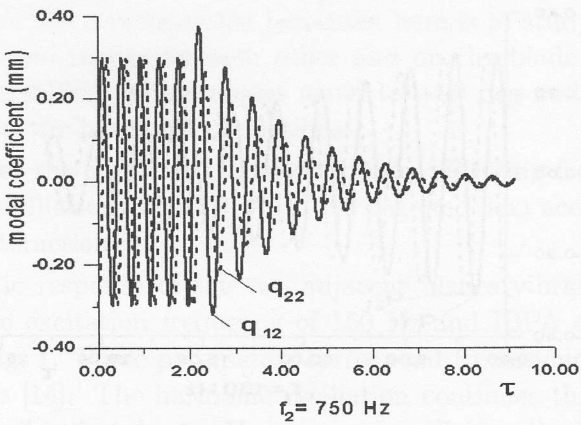

Figure 4. The aeroelastic oscillations of two adjacent blades corresponding to the second mode and IBPA equal −90 deg as the initial condition in the harmonic motion, $t = \tau/316$ s.

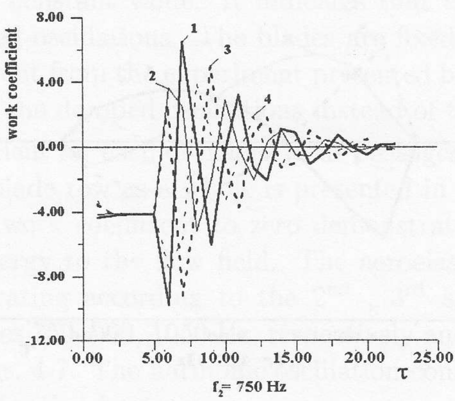

Figure 5. The work coefficient for each vibrating blade (2nd mode), and IBPA equal −90 deg as the initial condition in the harmonic motion, $t = \tau/316$ s.

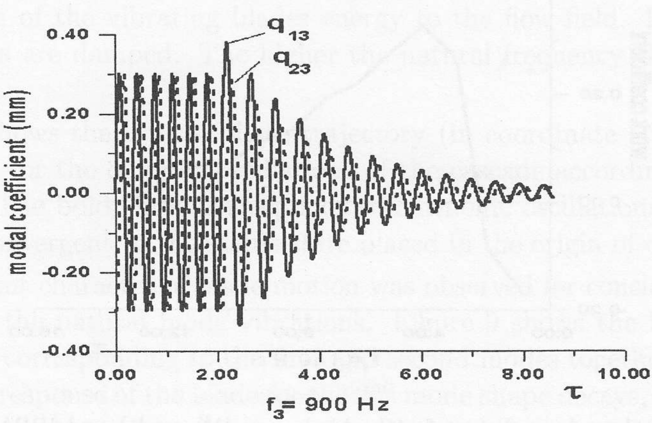

Figure 6. The aeroelastic oscillations of two adjacent blades corresponding to the third mode and IBPA −90 deg as the initial condition in the harmonic motion, $t = \tau/316$ s.

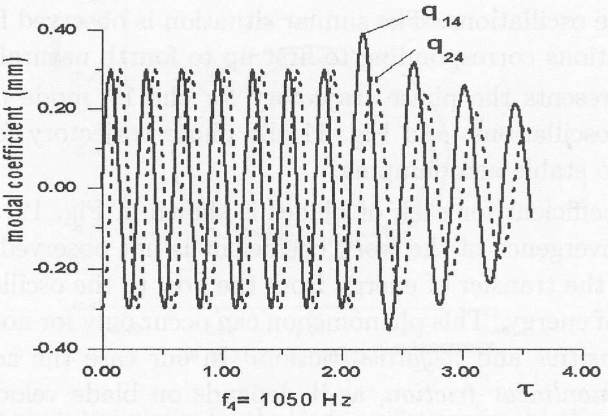

Figure 7. The aeroelastic oscillations of two adjacent blades corresponding to the fourth mode, and IBPA −90 deg as the initial condition in the harmonic motion, $t = \tau/316$ s.

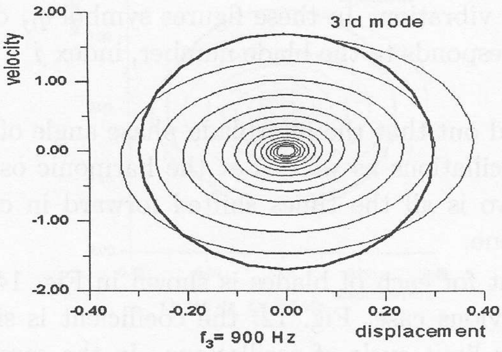

Figure 8. The phase trajectory for the blade oscillations according to the 3rd mode, and IBPA equal −90 deg as the initial condition in the harmonic motion, $t = \tau/316$ s.

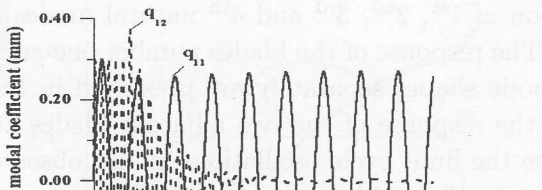

Figure 9. The aeroelastic oscillations of two adjacent blades corresponding to the 1st and 2nd mode, and IBPA −90 deg as the initial condition in the harmonic motion, $t = \tau/316$ s.

to the limit cycle oscillations. The similar situation is observed for coupled fluid-structure oscillations corresponding to first up to fourth natural modes.

Figure 10 presents the phase trajectory for the 1^{st} mode corresponding to the limit cycle oscillations, and Fig. 11 the phase trajectory for the 2^{nd} mode corresponding to stable aerodamping.

The work coefficient for each of blades is shown in Fig. 12. In this case the monotonous convergence of the work coefficient is not observed. It corresponds alternatively to the transfer of energy from the flow to the oscillating blades and the dissipation of energy. This phenomenon can occur only for nonlinear vibrating systems with *positive* and *negative* friction. In our case the aerodynamic load plays a role of *nonlinear friction*, as it depends on blade velocity. Equality of both positive and negative energy leads to the limit cycle of oscillations.

Figure 13 shows the blade response of oscillations corresponding to the first, second and third modes together for IBPA equal to −90 deg as the initial condition in the harmonic vibration. In these figures symbol q_{ij} denotes the modal coefficient, index i corresponds to the blade number, index j – to the mode number.

It should be pointed out that the interblade phase angle of −90 deg remains constant at coupled oscillations as well as at the harmonic oscillation. The response of the blade two is all the times shifted forward in comparison to the response of the blade one.

The work coefficient for each of blades is shown in Fig. 14. In this case, in comparison to the previous case, Fig. 12, the coefficient is similar for each of blades and leads to the limit cycle of oscillations. In the case of the blade row the shape of the function is different, so the influence of the interaction of the modes is visible.

Figures 15-18 show the two adjacent blades response in the case when considered is interaction of 1^{st}, 2^{nd}, 3^{rd} and 4^{th} natural modes at the initial IBPA equal to −90 deg. The response of the blades number one and two corresponding to one to fourth mode shapes separately are presented in Figs. 15, 16. Figures 17 and 18 present the response of the two adjacent blades at the coupled oscillations. In this case the limit cycle oscillations is also observed. The reponse of the blades show no significient differences so the influence of the higher modes on the response of the blade and the work coefficient is not so important.

Numerical results of the coupled fluid-structure aeroelastic calculations according to the first four blade modes oscillations at the initial interblade phase angle of +90 deg are presented in Figs. 19-35. The aeroelastic response of the two adjacent blade vibrating according to the 1^{st} mode, the excitation frequency of 150 Hz and IBPA equal to +90 deg is presented in Fig. 19. These parameters corresponds to positive aerodamping coefficients (the damping condition see

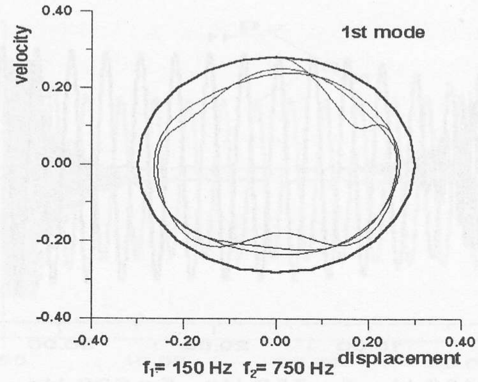

Figure 10. The phase trajectories for the blade oscillations for the 1st and 2nd modes.

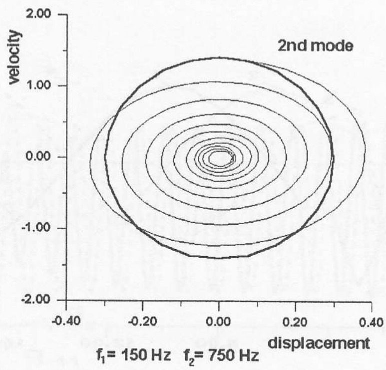

Figure 11. The phase trajectories for the blade oscillations for the 1st and 2nd modes.

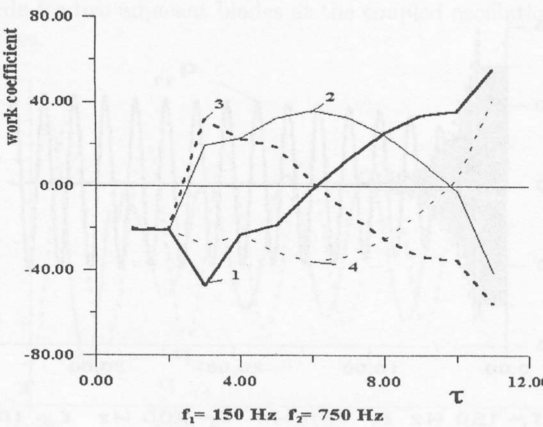

Figure 12. The work coefficient for each blade (1st and 2nd modes).

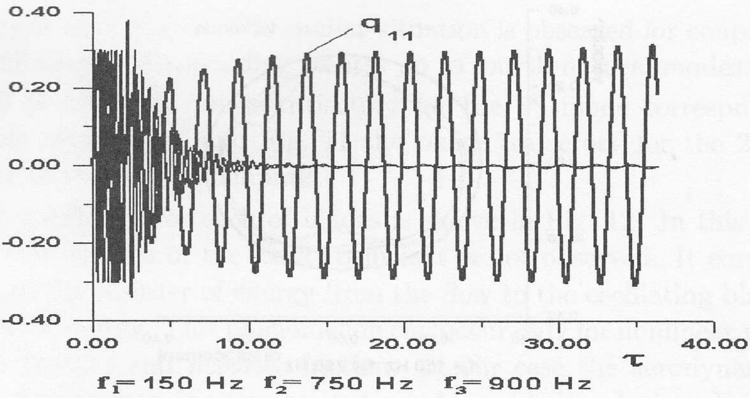

Figure 13. The blade response at the coupled oscillations for the 1st, 2nd and 3rd modes.

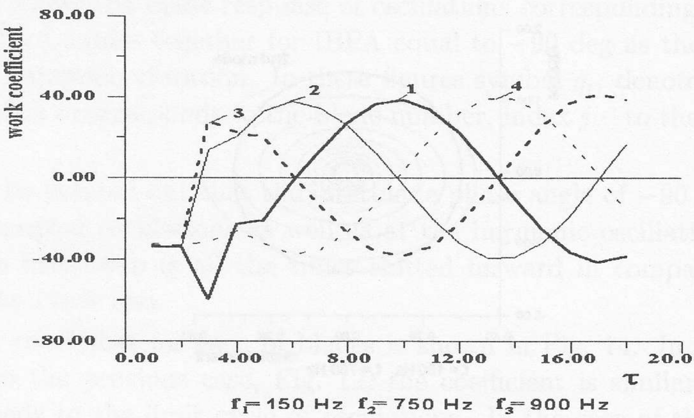

Figure 14. The work coefficient for each vibrating blade (1st, 2nd and 3rd modes).

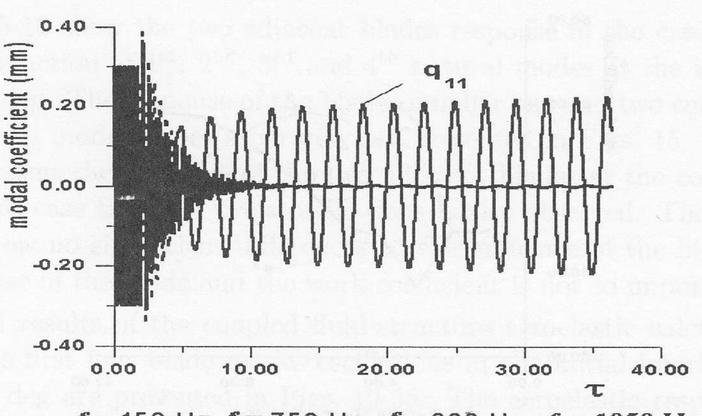

Figure 15. The first blade oscillations for the 1st, 2nd, 3rd and 4th modes (IBPA of −90 deg), $t = \tau/316$ s.

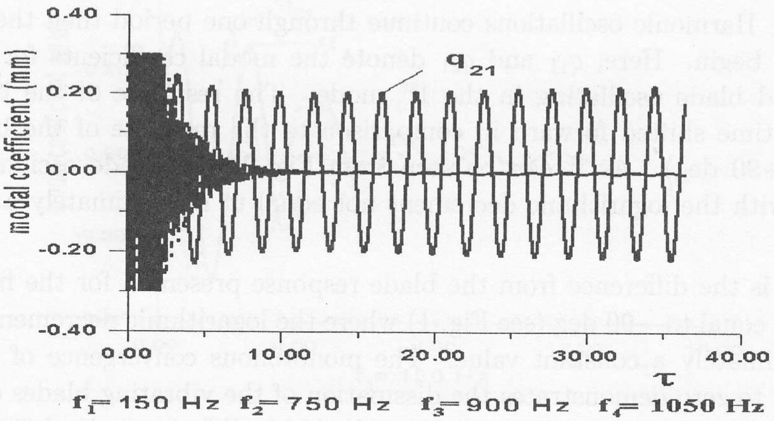

Figure 16. The second blade oscillations for the 1st, 2nd, 3rd and 4th modes (IBPA of −90 deg) $t = \tau/316$ s.

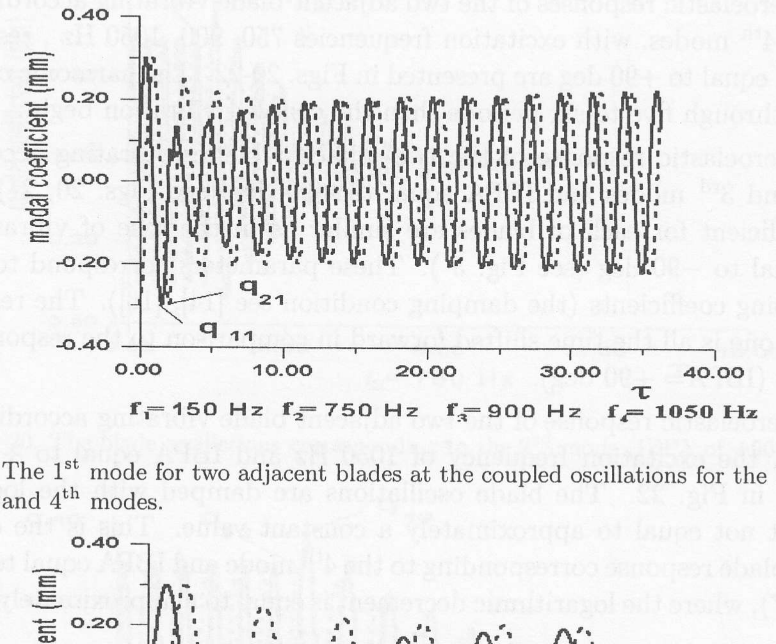

Figure 17. The 1st mode for two adjacent blades at the coupled oscillations for the 1st, 2nd, 3rd and 4th modes.

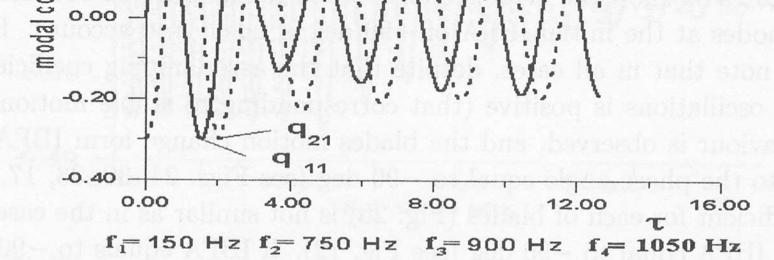

Figure 18. The 1st mode for two adjacent blades at the coupled oscillations for the 1st, 2nd, 3rd and 4th modes.

[14], [16]). Harmonic oscillations continue through one period then the coupled vibrations begin. Here, q_{11} and q_{21} denote the modal coefficients for the first and second blade oscillating in the 1^{st} mode. The response of the blade one is all the time shifted forward in comparison to the response of the blade two (IBPA= +90 deg). As it can be seen from Fig. 19 the blade oscillations are damped with the logarithmic decrement not equal to approximately a constant value.

There is the difference from the blade response presented for the first mode and IBPA equal to −90 deg (see Fig. 1) where the logarithmic decrement is equal to approximately a constant value. The monotonous convergence of the work coefficient to zero demonstrates the dissipation of the vibrating blades energy to the flow field. The way of dissipation is a little bit different as in the case of 1^{st} mode vibration with IBPA equal to −90 deg (see Figs. 2, 3).

The aeroelastic responses of the two adjacent blade vibrating according to the 2^{nd} , 3^{rd}, 4^{th} modes, with excitation frequencies 750, 900, 1050 Hz , respectively and IBPA equal to +90 deg are presented in Figs. 20-22. The harmonic oscillation continue through five to six periods then the coupled vibration begin.

The aeroelastic response of the two adjacent blades vibrating according to the 2^{nd} and 3^{rd} modes and IBPA equal to +90 deg (see Figs. 20, 21) and the work coefficient for each of blades are similar as in the case of vibration with IBPA equal to −90 deg (see Fig. 5). These parameters correspond to positive aerodamping coefficients (the damping condition see [14], [16]). The response of the blade one is all the time shifted forward in comparison to the response of the blade two (IBPA= +90 deg).

The aeroelastic response of the two adjacent blade vibrating according to the 4^{th} mode, the excitation frequency of 1050 Hz and IBPA equal to +90 deg is presented in Fig. 22. The blade oscillations are damped with the logarithmic decrement not equal to approximately a constant value. This is the difference with the blade response corresponding to the 4^{th} mode and IBPA equal to −90 deg (see Fig. 7), where the logarithmic decrement is equal to a approximately constant value.

Figures 23-32 show the blade response when the interaction between different natural modes at the initial IBPA of +90 deg is taken into account. It is interesting to note that in all cases, despite that the aerodamping coefficient at the harmonic oscillations is positive (that corresponding to stable motion), a transient behaviour is observed, and the blades motion change form IBPA equal to +90 deg to the phase angle equal to −90 deg (see Figs. 24, 30, 33, 17, 18). The work coefficient for each of blades (Fig. 26) is not similar as in the case of vibration with IBPA equal to −90 deg (see Fig. 12). If IBPA equals to −90 deg then it represents an unstable condition for the first mode, the amplitude of the first

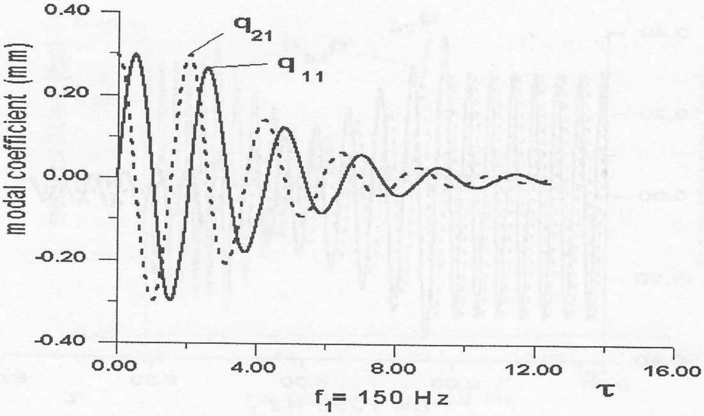

Figure 19. The adjacent blade oscillations corresponding to the 1st mode (IBPA of +90 deg).

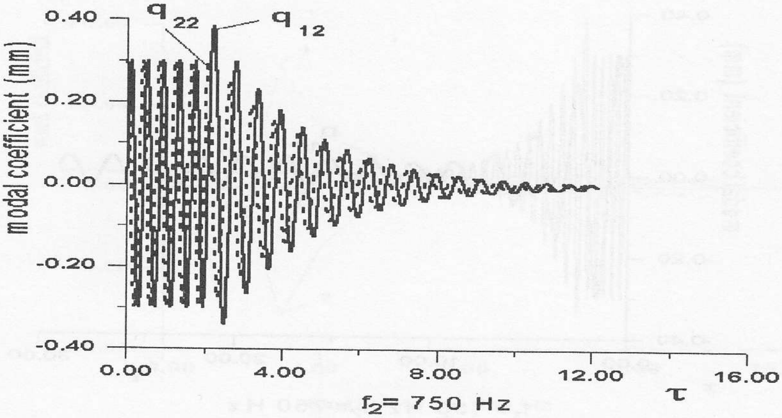

Figure 20. The blade oscillations corresponding to the 2nd mode (IBPA of +90 deg).

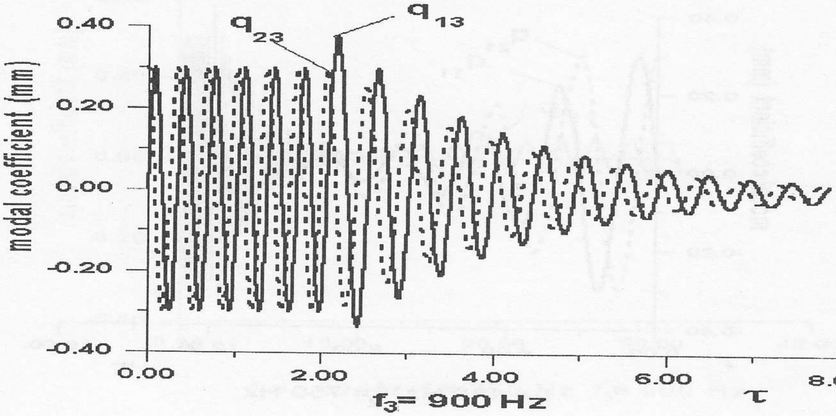

Figure 21. The blade oscillations corresponding to the 3rd mode (IBPA of +90 deg).

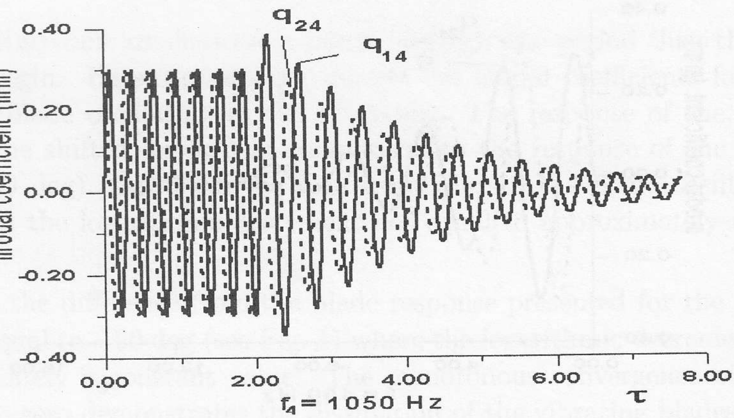

Figure 22. The blade oscillations corresponding to the 4th mode (IBPA of +90 deg).

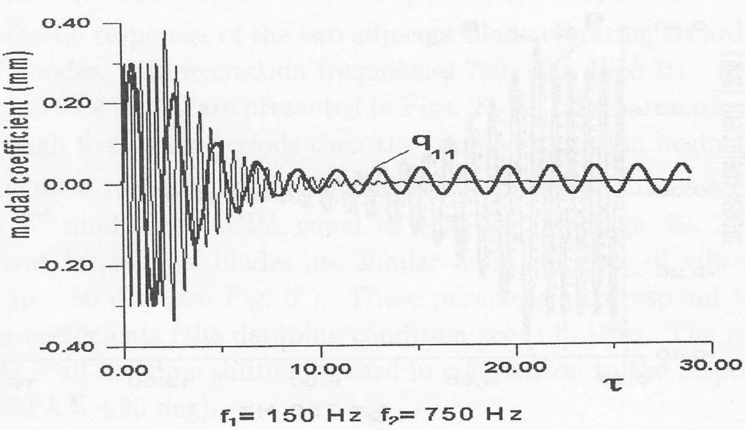

Figure 23. The blade oscillations corresponding to the 1st and 2nd modes (IBPA of +90 deg).

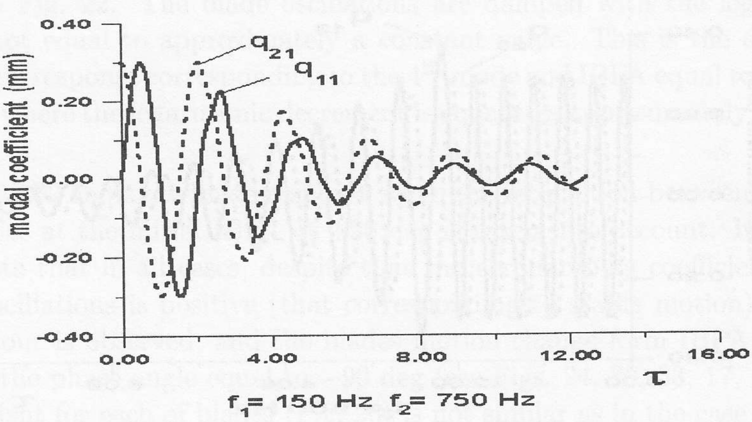

Figure 24. The blade oscillations corresponding to the 1st and 2nd modes (IBPA of +90 deg).

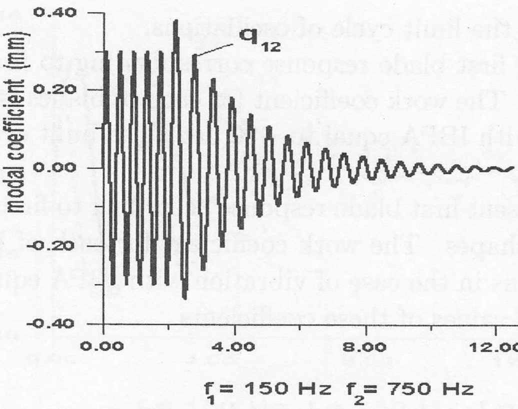

Figure 25. The 2[nd] mode at the blade oscillations for the 1[st] and 2[nd] modes (IBPA of +90 deg).

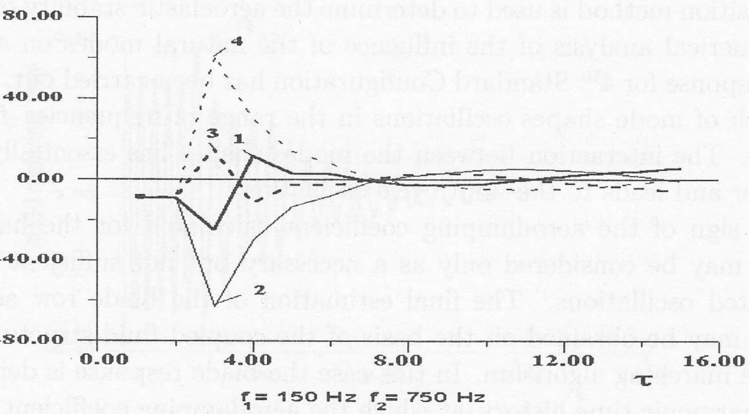

Figure 26. The work coefficient for blades for the 1[st] and 2[nd] mode (IBPA of +90 deg).

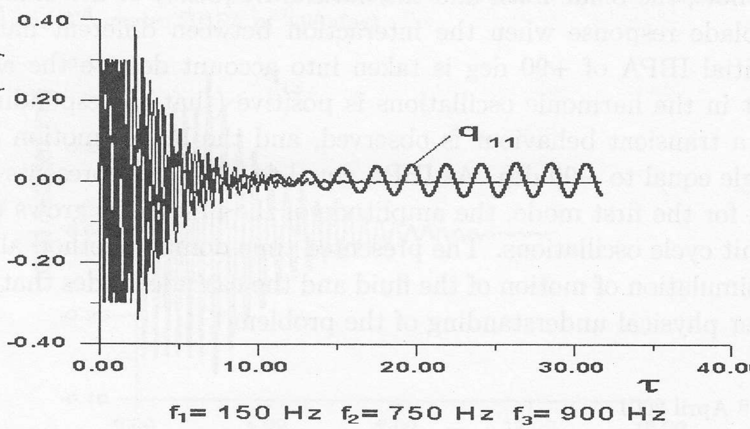

Figure 27. The blade oscillations for the 1[st], 2[nd] and 3[rd] modes (IBPA of +90 deg).

mode grows approaching the limit cycle of oscillations.

Figures 28-30 present first blade response corresponding to first, second and third blade mode shapes. The work coefficient for each of blades is not similar as in the case of vibration with IBPA equal to −90 deg. The limit cycle oscillations is observed.

Figures 32, 29, 30 present first blade response according to first, second, third and fourth blade mode shapes. The work coefficient for each of blades (similar to Fig. 31) is not similar as in the case of vibration with IBPA equal to −90 deg. The main difference is in values of these coefficients.

5 Conclusions

In the present study, the simultaneous time domain method and the modal superposition method is used to determine the aeroelastic stability of the cascade. The numerical analysis of the influence of the natural modes on an aeroelastic blade response for 4^{th} Standard Configuration has been carried out. It has shown that each of mode shapes oscillations in the range of frequencies $f > 150$ Hz is damped. The interaction between the modes shapes has essentially a nonlinear character and leads to the limit cycle oscillations.

The sign of the aerodamping coefficient calculated for the harmonic oscillations, may be considered only as a necessary but not sufficient condition for self-excited oscillations. The final estimation of the blade row aeroelastic behaviour may be obtained on the basis of the coupled fluid-structure solution in the time marching algorithm. In this case the blade response is defined not only by the harmonic time history, at which the aerodamping coefficient has been calculated, but also by such parameters which influence the aerodynamic force as the mass flow, the blade mass and the natural frequency of the blade.

The blade response when the interaction between different natural modes at the initial IBPA of +90 deg is taken into account despite the aerodamping coefficient in the harmonic oscillations is positive (that corresponding to stable motion), a transient behaviour is observed, and the blades motion change to a phase angle equal to −90 deg. As IBPA equal to −90 deg represents an unstable condition for the first mode, the amplitude of the first mode grows approaching to the limit cycle oscillations. The presented time domain method allows a more realistic simulation of motion of the fluid and the cascade blades that should lead to a better physical understanding of the problem.

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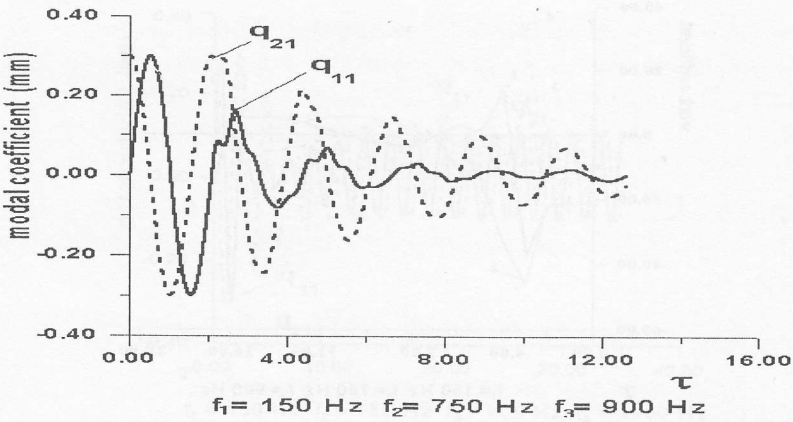

Figure 28. The first and second blade oscillations corresponding to 1st mode for 1st, 2nd and 3rd modes (IBPA of +90 deg).

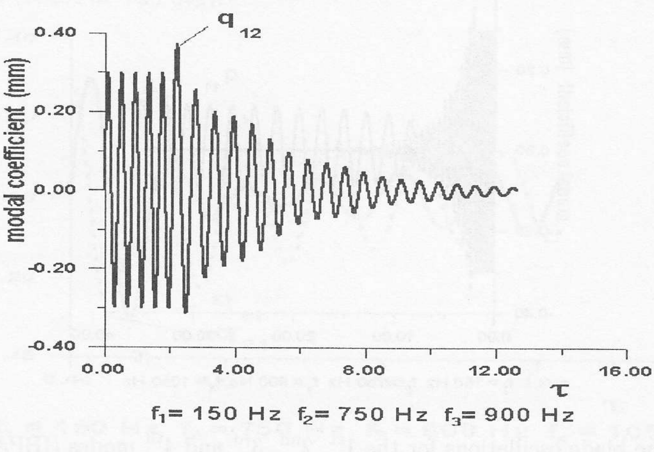

Figure 29. The first blade responses corresponding to 2nd mode at the blade oscillations for 1st, 2nd and 3rd modes (IBPA of +90 deg).

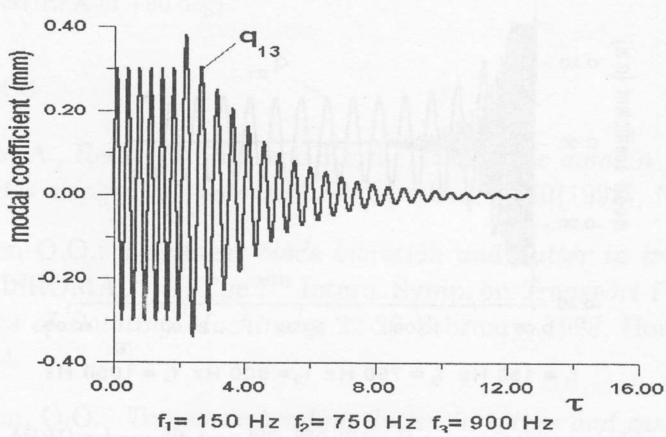

Figure 30. The first blade response corresponding to 3rd mode at the blade oscillations for 1st, 2nd and 3rd modes (IBPA of +90 deg).

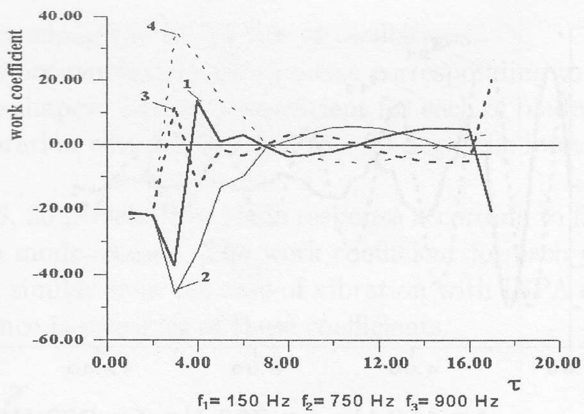

Figure 31. The work coefficient for blades for 1st, 2nd and 3rd modes (IBPA of +90 deg).

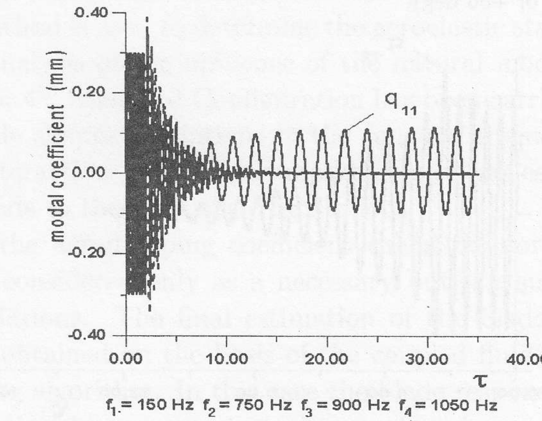

Figure 32. The blade oscillations for the 1st, 2nd, 3rd and 4th modes (IBPA of +90 deg).

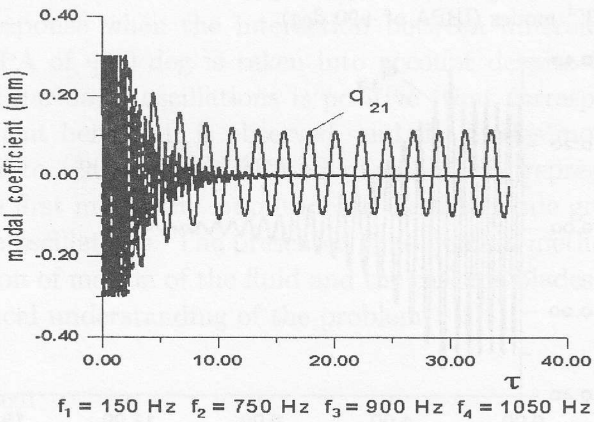

Figure 33. The blade oscillations for the 1st, 2nd, 3rd and 4th modes (IBPA of +90 deg).

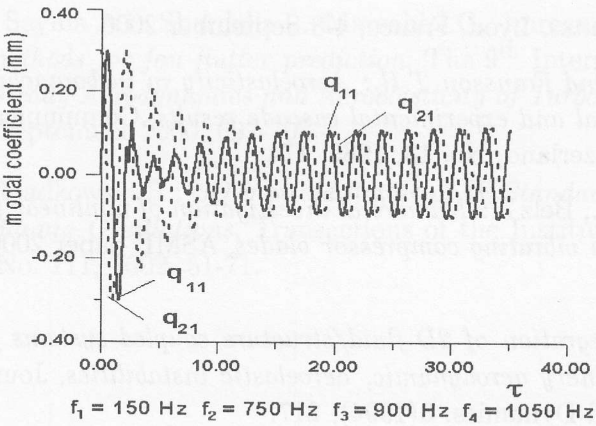

Figure 34. The 1st mode for two adjacent blades at the oscillations in the 1st, 2nd, 3rd and 4th modes (IBPA of +90 deg).

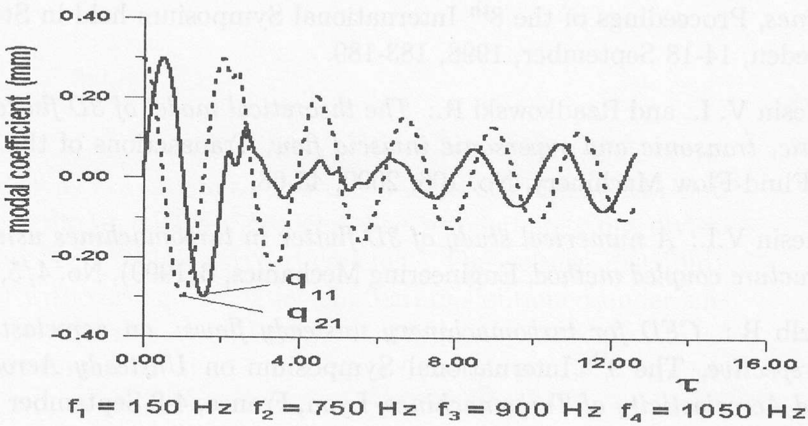

Figure 35. The 1st mode for two adjacent blades at the oscillations in the 1st, 2nd, 3rd and 4th modes(IBPA of +90 deg).

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