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TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

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The laminar flow of liquids in a flat-wall face clearance with a variable width between stationary and rotating rings

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Abstract

The paper deals with the laminar flow through a flat-wall face clearance with a variable width between the stationary and rotating rings. The flow through such an clearance with immobile walls is described, too. Besides the longitudinal clearance, the face clearance is an essential structural element of hydraulic machines. A characteristic example is the face clereance between the slip and stopper ring the balance disk in a multistage centrifugal pump. In the case of conventional solutions of the balance disc the structionally required width of the clearance leads to a turbulent flow of the liquid through the clearance [2]. If the outer diameter of the balance disc is reduced, the pressure under the disc increases, the width of the face clearance gets smaller and the flow rate through this clearance become less intensive. In result, the turbulence of the flow drops and it becomes possible to enter the zone of transient and laminar flows [7]. The reduction of the diameter of the balance disc is connected with a reduction of leakage losses and friction resistance, and thus the pump becomes more efficient. In the case of applying a self-aligning disc, the momentum of the pressure forces may deviate it, preventing a dry friction of the slip-ring against the stopper ring. Such a disc can operate with a considerably narrower clearance than that required by stiff discs. Laboratory-scale investigations carried out on a model of a balance disc with a self-aligning support of the rotating ring have confirmed that in the case of a centrifugal flow the distribution of pressure in the face clearance prevents the occurrance of dry friction, i.e. a contact of both rings [7, 8]. In convectional structures of balance discs this effect is neutralized by their stiffness, so that the possibility of dry friction cannot be prevented.

Keywords: Laminar flow; Face clearance

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Nomenclature

a, a_m	– width of the clearance, mean value of this width, m
F, F_a	– force, axial force, N
M, M_x, M_y	– momentum, components of the momentum vector, Nm
p, p_1, p_2	– pressure, pressure on the internal radius r_1, pressure on the external radius r_2 of the clearance, Pa
Q	– rate of flow through the clearance, m^3/s
r, φ, z	– coordinates of the cylindrical coordinate system, m, rad, m
r_1, r_2	– internal radius, external radius of the clearance, m
Re, Re_P, Re_C	– Reynolds similarity number, Re in Poiseuille's flow, Re in Couette's flow
$u = \omega r$	– circumferential speed of then wall, /ms
v_r, v_φ, v_z	– components of the vector of the flow rate in a cylindrical coordinate system, m/s
x, y	– coordinates of a rectangular the coordinate system, m
α	– angle of convergence of the walls of the clearance, rad
ε	– ratio of the length of the face clearance to its internal radius
μ	– viscosity, kg/ms
$\nu = \mu/\rho$	– kinematic viscosity, m^2/s
ω	– angular velocity of the rotating ring, s^{-1}
ρ	– density of the liquid, kg/m^3
ψ	– coefficient of the axial force

1 Equations of motion

A diagram of a flat-walled face clearance with a variable width as well as its characteristic dimensions are to be seen in Fig. 1.

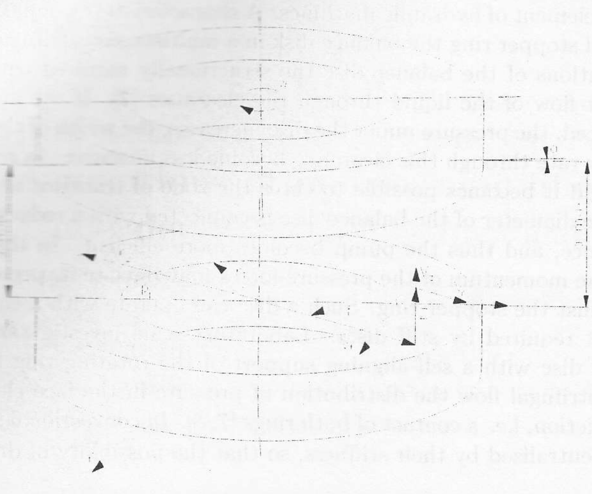

Figure 1. Diagram of a flat-walled face clearance with varying widths.

The flow has been assumed to be stationary, and so the influence of the body forces could be neglected. Moreover it has been assumed that along the width of the clearance the pressure is constant, as a laminar flow in a narrow clearance is fully developed as two combined boundary layers, in which the pressure gradient transverse to the flow is one order of magnitude smaller than the gradient in the direction of the flow [6, 15]. Basing on such assumptions the motion of liquid in the clearance is described by the $N-S$ equations, which in the cylindrical coordinate system yield

$$v_r\frac{\partial v_r}{\partial r}+\frac{v_\varphi}{r}\frac{\partial v_r}{\partial \varphi}-\frac{v_\varphi^2}{r}=$$
$$=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 v_r}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \varphi^2}+\frac{\partial^2 v_r}{\partial z^2}+\frac{1}{r}\frac{\partial v_r}{\partial r}-\frac{2}{r^2}\frac{\partial v_\varphi}{\partial \varphi}-\frac{v_r}{r^2}\right), \quad (1)$$

$$v_r\frac{\partial v_\varphi}{\partial r}+\frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial \varphi}-\frac{v_r v_\varphi}{r}=$$
$$=-\frac{1}{\rho r}\frac{\partial p}{\partial \varphi}+\nu\left(\frac{\partial^2 v_\varphi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 v_\varphi}{\partial \varphi^2}+\frac{\partial^2 v_\varphi}{\partial z^2}+\frac{1}{r}\frac{\partial v_\varphi}{\partial r}-\frac{2}{r^2}\frac{\partial v_r}{\partial \varphi}-\frac{v_\varphi}{r^2}\right), \quad (2)$$

$$0=\frac{\partial p}{\partial z}. \quad (3)$$

In the case of these assummptions the equation of continuity takes the following form

$$\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_\varphi}{r\partial \varphi}=0. \quad (4)$$

The face clearance between the slip and the stoping rings of the balance disc is characterized by the ratios of its functional dimensions and the width; in the case of a self-aligning disc these may assume the following values: $a = 0.12$ to 0.04 mm, $r_1 = 50$ to 100 mm, $\varepsilon = r_2 - r_1/r_1 = 0.14$ to 0.36.

From these dimensions the following proportions may be derived: $a/r_1 = 0.0004$ ti 0.0016 oraz $\varepsilon^4 = 0.000386$ to 0.00168.

In other words, assessing the order of values of the respective terms of the Eqs. (1) and (2), the ratio a/r_1 may be replaced by ε^4.

Subsequently we convert Eqs. (1) and (2) into dimensionless quantities, introducing the following substitutes [10]

$$v_r = \omega r_1 \bar{v}_r,$$
$$v_\varphi = \omega r_1 \bar{v}_\varphi,$$
$$r = r_1\bar{r} = r_1(1+\varepsilon),$$
$$r\varphi = r_1\bar{r}\varphi,$$
$$z = a\bar{z} = \varepsilon^4 r_1 \bar{z}.$$

The pressure p is expressed by a dimensionless quantity basing on the relation

$$\frac{\partial p}{\partial r} \approx \mu \frac{\partial^2 v_r}{\partial z^2}.$$

Substituting dimensionless quantities we get

$$\frac{\partial p}{\varepsilon r_1 \partial \bar{x}} = \frac{\mu \omega r_1}{\varepsilon^8 r_1^2} \frac{\partial^2 \bar{v}_r}{\partial \bar{z}^2},$$

and hence: $p = \frac{\mu\omega}{\varepsilon^7}\bar{p}$.
We also introduce the Reynolds number

$$\mathrm{Re} = \omega r_1 a/\nu = \varepsilon^4 \omega r_1^2/\nu.$$

Having substituted the dimensionless quantities in the Eqs. (1), and (2) we have

$$\mathrm{Re}\varepsilon^3\left(\bar{v}_r\frac{\partial \bar{v}_r}{\partial \bar{x}} + \varepsilon\frac{\bar{v}_\varphi}{\bar{r}}\frac{\partial \bar{v}_r}{\partial \varphi} - \varepsilon\frac{\bar{v}_\varphi^2}{\bar{r}}\right) =$$
$$= \frac{\partial \bar{p}}{\partial \bar{x}} + \left(\varepsilon^6\frac{\partial^2 \bar{v}_r}{\partial \bar{x}^2} + \varepsilon^8\frac{1}{\bar{r}^2}\frac{\partial^2 \bar{v}_r}{\partial \varphi^2} + \frac{\partial^2 \bar{v}_r}{\partial \bar{z}^2} + \varepsilon^7\frac{1}{\bar{r}}\frac{\partial \bar{v}_r}{\partial \bar{x}} - \varepsilon^8\frac{2}{\bar{r}^2}\frac{\partial \bar{v}_\varphi}{\partial \varphi} - \varepsilon^8\frac{\bar{v}_r}{\bar{r}^2}\right),$$

$$\mathrm{Re}\varepsilon^3\left(\bar{v}_r\frac{\partial \bar{v}_\varphi}{\partial \bar{x}} + \varepsilon\frac{\bar{v}_\varphi}{\bar{r}}\frac{\partial \bar{v}_\varphi}{\partial \varphi} + \varepsilon\frac{\bar{v}_r\bar{v}_\varphi}{\bar{r}}\right) =$$
$$= -\frac{\varepsilon}{1+\varepsilon\bar{x}}\frac{\partial \bar{p}}{\partial \varphi} + \left(\varepsilon^6\frac{\partial^2 \bar{v}_\varphi}{\partial \bar{x}^2} + \varepsilon^8\frac{1}{\bar{r}^2}\frac{\partial^2 \bar{v}_\varphi}{\partial \varphi^2} + \frac{\partial^2 \bar{v}_\varphi}{\partial \bar{z}^2} + \varepsilon^7\frac{1}{\bar{r}}\frac{\partial \bar{v}_\varphi}{\partial \bar{x}} + \varepsilon^8\frac{2}{\bar{r}^2}\frac{\partial \bar{v}_r}{\partial \varphi} - \varepsilon^8\frac{\bar{v}_\varphi}{\bar{r}^2}\right).$$

2 Calculation of pressure distribution in the clearance by applying equations of motion, simplified to expressions of the order $> \varepsilon / (1 + \bar{x}\,\varepsilon)$

If in the Eqs. (1) and (2) the expressions of the order $\varepsilon/(1+\bar{x}\varepsilon)$ and less are omitted, it will result from the derived dimensionless equations that they have been reduced to:

$$\frac{\partial p}{\partial r} - \mu\frac{\partial^2 v_r}{\partial z^2} = 0, \tag{1a}$$

$$\mu\frac{\partial^2 v_\varphi}{\partial z^2} = 0. \tag{2a}$$

Moreover, the following boundary conditions have been assumed

for $z = 0$, we get: $v_r = 0$ and $v_\varphi = 0$,

for $z = a$, we get: $v_r = 0$ and $v_\varphi = \omega r$,
for $r = r_1$, we get: $p = p_1$,
for $r = r_2$, we get: $p = p_2$.

From such a geometry of the clereance results the possibility to determine its width by means of the following formula:

$$a = a_m + \alpha r \cos\varphi \tag{5}$$

in which:

$$a_m = \frac{a_{min} + a_{max}}{2}. \tag{6}$$

Integrating the Eqs. (1) and (2) versus the variable z and taking into account the relation (3) as well as the assumed boundary conditions, we get

$$v_r = \frac{1}{2\mu}\frac{\partial p}{\partial r}(z^2 - az), \tag{7}$$

$$v_\varphi = r\omega\frac{z}{a}. \tag{8}$$

Substituting the velocity components expressed by the formulae (7) and (8) and their derivations into Eq. (4) and applying the parametric equations:

$$b = \alpha\cos\varphi, \tag{9}$$

$$c = 6\mu\omega\alpha\sin\varphi \tag{10}$$

and substituting also the relation (5), we get a heterogeneous differential equation of the second order with variable coefficients

$$r(a_m + br)^3\frac{\partial^2 p}{\partial r^2} + (4b^3r^3 + 9a_m b^2 r^2 + 6a_m^2 br + a_m^3)\frac{\partial p}{\partial r} + cr^2 = 0. \tag{11}$$

After such simplifications the integration of Eq. (11) permits to calculate the pressure in any arbitrary point of the face clearance. Eq. (11) can be solved both analytically and numerically after it has been reduced to a differentiating form.

2.1 Analytical solution

Applying the substitution method we get a differential equation of the first order. It can be solved by finding first the general integral of the corresponding homogenous equation and then applying the method of the variations of constants. As a result of integration the course of pressure in the clearance along the radius r was determined for the assumed angle φ

$$p(r,\varphi) = \frac{-2a_m^3 c + 3b^3 C_1}{3a_m^2 b^3(a_m + br)} + \\ + \frac{a_m^3 c + 3b^3 C_1}{6a_m b^3(a_m + br)^2} + C_2 + \frac{C_1}{a_m^3}\ln r + \frac{-a_m^3 c - 3b^3 C_1}{3a_m^3 b^3}\ln(a_m + br). \tag{12}$$

After the substitution of the relations (9) and (10), Eq. (12) contains discontinuities in the case of $\varphi = \pm\pi/2$. Therefore, it must be solved by substituting the angle φ in the intervals: $-\pi/2 < \varphi < \pi/2$ and $\pi/2 < \varphi < 3\pi/2$. The final result is obtained by joining the solutions concerning both these intervals.

The integration constants C_1 and C_2 can be determined basing on the boundary conditions, which again are determined by the values of the pressure on both sides of the clearance: for a centrifugal flow $p_1 > p_2$, and for a centripetal flow $p_1 < p_2$.

2.2 Numerical solution

The numerical solution of Eq. (11) simplifies somewhat further calculations of the forces and moments, which in the case of applying the analitical solution of this equation also require discretisation. For a given angle φ_i the radius between the inlet and the outlet of the clearance ought to be divided into N sections with the length Δr. Then the pressure at the point determined by the angle φ_i and the radius r_j, can be calculated by means of Eq. (11) in the differentiating form

$$p_j = \frac{p_{j+1} + p_{j-1}}{2} + \frac{4b^3r^3 + 9a_m b^2 r^2 + 6a_m^2 br + a_m^3}{(a_m + br)^3}\frac{\Delta r}{r}\frac{p_{j+1} - p_{j-1}}{4} + \frac{cr\Delta r^2}{2(a_m + br)^3} \tag{11a}$$

in which the pressure on the radius r_1 amounts to $p_{j=1}$, the pressure on the radius r_2 amounts to $p_{j=K+1}$, and the Eqs. (9) and (10) determine the parameters b and c for the angle φ_i.

3 Calculation of the pressure in the face clearance by means of equations of motion simplified to expressions of the order $> \varepsilon^3$ (taking into account circumferential stresses)

The flow dealt with in this paper may be described in more detail leaving out the expressions of the order ε^3 or smaller in the equations (1) and (2). Then besides the Eqs. (1a), (3) and (4) we get Eq. (2) in the following form

$$-\frac{\partial p}{r\partial\varphi} + \mu\frac{\partial^2 v_\varphi}{\partial z^2} = 0. \tag{2b}$$

Moreover, similarly as before, the boundary conditions and geometry of the clereance were assumed.

Integrating Eq. (2b) versus the variable z and taking into account the relation (3) as well as the boundary conditions, we get

$$v_\varphi = \frac{1}{2\mu}\frac{\partial p}{r\partial\varphi}(z^2 - az) + r\omega\frac{z}{a}. \tag{8a}$$

Substituting the components of velocity expressed by the formulae (7) and 8a) and their derivatives into Eq. (4) and assuming the following coefficients:
$A = a_m/\alpha$,
$B = 6\mu\omega/\alpha^2$,
o be constant, we get a heterogeneous differential equation of the second order vith partial derivatives and variable coefficients

$$(A + r\cos\varphi)^3\frac{\partial^2 p}{\partial r^2} + \left[(A + r\cos\varphi)^3\frac{1}{r} + 3(A + r\cos\varphi)^2\cos\varphi\right]\frac{\partial p}{\partial r} +$$
$$+\frac{1}{r^2}(A + r\cos\varphi)^3\frac{\partial^2 p}{\partial\varphi^2} - \frac{3}{r}(A + r\cos\varphi)^2\sin\varphi\frac{\partial p}{\partial\varphi} + Br\sin\varphi = 0. \tag{13}$$

This equation can be solved approximately: analytically by means of serial xpansion or numerically.

As the Eq. (12) is discontinuos and an integral of Eq. (11), there may also ccur difficulties due to the lack of proofs of convergences in the serial expansion f Eq. (13). For this reason we solve the problem numerically.

This was done by applying a grid of points, dividing the circumference into K segments and the radius into N sections. In the case of points which are not ituated on the boundary of the region (interior points) differentiating equations vere set up, replacing the derivatives in Eq. (11a) by differentiating quotients. or the angle and the radius central differences of pressures were assumed. The esults were equations for $1 \le i \le K$ and $2 \le j \le N - 1$

$$(A + r_j\cos\varphi_i)^3\frac{p_{j+1,i} - 2p_{j,i} + p_{j-1,i}}{\Delta r_j^2} +$$
$$\left[(A + r_j\cos\varphi_i)^3\frac{1}{r_j} + 3(A + r_j\cos\varphi_i)^2\cos\varphi_i\right]\frac{p_{j+1,i} - p_{j-1,i}}{2\Delta r_j} +$$
$$+\frac{1}{r_j^2}(A + r_j\cos\varphi_i)^3\frac{p_{j,i+1} - 2p_{j,i} + p_{j,i-1}}{\Delta\varphi_i^2} -$$
$$-\frac{3}{r_j}(A + r_j\cos\varphi_i)^2\sin\varphi_i\frac{p_{j,i+1} - p_{j,i-1}}{2\Delta\varphi_i} + Br_j\sin\varphi_i = 0 \tag{13a}$$

here $K\Delta\varphi_i = 2\pi$.

Thus we get $K(N-2)$ equations, the number of which is equal to the pro-uct of KN unknown values. However, $2K$ lacking roots are determined by the boundary conditions at the inner and outer boundaries of the clearance:

$p_{j=1,i} = p_1$,

$p_{j=N,i} = p_2$.

Applying Eq. (13a) we can calculate subsequently the pressure in the respective points of the grid, expressed by the transformed formula

$$p_{j,i} = \frac{1}{2}\frac{\Delta r_j^2 r_j^2 \Delta\varphi_i^2}{r_j^2\Delta\varphi_i^2 + \Delta r_j^2}\left[\frac{p_{j+1,i}+p_{j-1,i}}{\Delta r_j^2} + \right.$$

$$+\frac{p_{j,i+1}+p_{j,i-1}}{r_j^2\Delta\varphi_i^2} + \left(\frac{1}{r_j} + \frac{3\cos\varphi_i}{A + r_j\cos\varphi_i}\right)\frac{p_{j+1,i}-p_{j-1,i}}{2\Delta r_j} -$$

$$\left. -\frac{3\sin\varphi_i}{A + r_j\cos\varphi_i}\frac{p_{j,i+1}-p_{j,i-1}}{2r_j\Delta\varphi_i} + \frac{Br_j\sin\varphi_i}{(A + r_j\cos\varphi_i)^3}\right]. \tag{13b}$$

4 Velocity of liquid flow in the clearance

The radial velocity v_r and circumferential velocity v_φ taken into account in our analysis of the flow rate of a liquid in the clearance can be calculated leaving in the Eqs. (1-4) those expressions which are larger than $\varepsilon/(1 + \bar{x}\varepsilon)$ or more precisely leaving those expressions that are larger than ε^3.

The radial velocity in the clearance can be calculated by means of Eq. (7). Having calculated the derivative of pressure making use of Eq. (12) and substituting it in Eq. (7), we obtain

$$v_r = \frac{1}{2\mu}\left[\frac{C_1}{a_m^3 r} - \frac{a_m^3 c + 3b^3 C_1}{3a_m b^2(a_m + br)^3}\right.$$

$$\left. -\frac{-2a_m^3 c + 3b^3 C_1}{3a_m^2 b^2(a_m + br)^2} + \frac{-a_m^3 c - 3b^3 C_1}{3a_m^3 b^2(a_m + br)}\right][z^2 - (a_m + br)z]. \tag{14}$$

Next, taking into account the parametric Eqs. (9) and (10) we get the velocity v_r at any arbitrary point of the clearance.

Velocity of flow of liquid in the clearance can also be calculated introducing into Eq. (7) the pressure gradient calculated iteratively by means of Eq. (11a).

In the case of more accurate calculations, the pressure gradient calculated iteratively by means of Eq. (13b) must be introduced into Eq. (7).

5 Rate of flow through the clearance

The flow rate Q through the clearance can be calculated, for instance, for the radius r_1, from the formula

$$Q = \int_0^a \int_0^{2\pi} v_r r_1 dz d\varphi = \frac{r_1}{12\mu} \int_0^{2\pi} (a_m + \alpha r_1 \cos\varphi)^3 \frac{dp(r_1, \varphi)}{dr} d\varphi \tag{15}$$

where the derivative $\frac{dp(r_1,\varphi)}{dr}$ is substituted similarly as in Eq. (14). Due to the necessity of neglecting the points of discontinuity, determined by $\varphi = \pm\pi/2$, we must divide the integrations in two ranges. Due to difficulties connected with the analytical solution of Eq. (14), also the solution of the approximated Eq. (15) seems to be justified. The velocity at the subsequent points of the clearance can be calculated by means of formula (14) or by means of iteration.

We may assume with a fairly good accuracy that the flow through the clearance shown in Fig. 1 equals to the flow through a clearance with a constant a_m, the remaining dimensions and the difference of pressure $p_1 - p_2$ being the same, as has been proved by the analysis presented in [10] and by experimental investigations dealt with in [4].

In the case of such an assumption the volumetric flow rate can be calculated by means of the transformed formula put forward by Huhn [14]

$$Q = [p_1 - p_2 + \frac{3\rho\omega^2}{20}(r_2^2 - r_1^2)] \frac{\pi a_m^3}{6\mu \ln(r_2/r_1)} \tag{16}$$

6 Axial force and the momentum caused by pressure in the clearance

The whole axial force exerted on the stopper ring or slip-rings is the result of the pressure inside the clearance. The force of this pressure amounts to

$$F_a = F_{a2} + F_{a1} = \int_{r_1}^{r_2} \int_{\varphi > -\pi/2}^{\varphi < \pi/2} p(r,\varphi) r dr d\varphi + \int_{r_1}^{r_2} \int_{\varphi > \pi/2}^{\varphi < 3\pi/2} p(r,\varphi) r dr d\varphi. \tag{17}$$

The asymetric distribution of pressure in the clearance leads to the occurence of momentum, whose projections on the axes x and y in the coordinate system of the assumed arrangement can be calculated making use of the following formulae

$$M_x = \int_{r_1}^{r_2} \int_{\varphi > -\pi/2}^{\varphi < \pi/2} p(r,\varphi) r^2 \sin\varphi dr d\varphi + \int_{r_1}^{r_2} \int_{\varphi > \pi/2}^{\varphi < 3\pi/2} p(r,\varphi) r^2 \sin\varphi dr d\varphi, \tag{18}$$

$$M_y = \int_{r_1}^{r_2} \int_{\varphi > -\pi/2}^{\varphi < \pi/2} p(r,\varphi) r^2 \cos\varphi dr d\varphi - \int_{r_1}^{r_2} \int_{\varphi > \pi/2}^{\varphi < 3\pi/2} p(r,\varphi) r^2 \cos\varphi dr d\varphi. \tag{19}$$

The momentum M acting on the slip ring amounts to

$$M = \sqrt{M_x^2 + M_y^2}. \tag{20}$$

The angle between the direction of action of the vector of the momentum M and the axis x is calculated by means of the formula

$$\varphi_M = \text{arc tg}(M_y/M_x). \tag{21}$$

If $0 < \varphi_M < \pi$, the momentum M counteracts the contact of the slip-ring with the stopper ring, and if $\pi < \varphi_M < 2\pi$, it will tend to a contact of both these rings within the frame of the elasticity of the structure.

In spite of the division into regions, in which the points of discontinuity ($\varphi = \pm\pi/2$) have been neglected, the formulae (17-19) gives rise to complications, similarly to formulae (15), which necessitates as iterative integration. This can be done for concrete data.

The surface of the rings was divided into K segments corresponding to the angle $\Delta\varphi$, which in our calculations was assumed to be $\Delta\varphi = 1^o$. Each segment was divided into N elementary fields, all of them having the same width Δr. In the quoted example it has been assumed that $N = 100$. In order to assess the assumed angle and radius, the calculations were repeated with the purpose of a multiple division. The results of these calculations differed in the case of further places.

The sums of the products expressing the pressure forces exerted on the elementary fields were calculated by means of the equation

$$F_j = \sum_{i=1}^{N} p(r_i, \varphi_j) r_i, \Delta\varphi\Delta r$$

where $\varphi_j = j\Delta\varphi + \Delta\varphi/2$,
$r_i = r_1 + i\Delta r$,
$N\Delta r = r_1 - r_2$

which were then summed up for the entire surface of the ring. Thus, the entire axial force acting on the ring amounted to

$$F_a = \sum_{j=0}^{K} F_j \tag{17a}$$

where $K\Delta\varphi = 360^o = 2\pi$.

The coefficient of the axial force resulting from the excess pressure inside the clearance compared to the pressure p_2, can be calculated applying the formula

$$\psi = [F_a - p_2\pi(r_2^2 - r_1^2)]/(p_1 - p_2)\pi(r_2^2 - r_1^2). \tag{22}$$

In order to calculate the momentum acting on the ring, the subsequent sums of products had to be calculated

$$M_j = \sum_{i=1}^{N} p(r_i, \varphi_j) r_i^2 \Delta r \Delta\varphi.$$

The projections of the vector of the momentum on the coordinates were calculated as follows

$$M_x = \sum_{j=1}^{K} M_j \sin(\varphi_j), \tag{18a}$$

$$M_y = \sum_{j=1}^{K} M_j \cos(\varphi_j). \tag{19a}$$

The momentum M was calculated using formula (20). The angle φ_M between the axis x and the vector of the momentum was calculated by means of Eq. (21). For the purpose of calculating both the axial force and the momentum the values of pressures calculated by means of Eq. (12) or iteratively applying the formula taken from Eq. (12a) as well as (13b).

7 Character of the flow through the clearance

Eq. (1a) describes Poiseuille flow through a clearance, whereas the Eqs. (2a) and (2b) describe the Couette flow, both neglecting and taking into account the pressure gradient. Due to changes in the width of a clearance with flat walls in the aforesaid phenomenon we can detect elements of the Jeffery-Hamel flow [1].

The character of the flow can be determined basing on Reynolds number, which is in the case of the Poiseuille flow through the clearence expressed by the following formula [1]

$$\mathrm{Re}_P = \frac{2av_{r,m}}{\nu} \tag{23}$$

and in the case of the Couette flow by the formula:

$$\mathrm{Re}_C = \frac{av_{\varphi m}}{\nu} \tag{24}$$

where

$v_{r,m}$ – is the mean radial velocity,

$v_{\varphi,m}$ – the mean velocity in Couette's flow (without the pressure gradient it equals half the peripheral speed u of the wall).

Laminar flows in a clearance are limited by critical Re numbers. In the case of Poiseuille's flow $\mathrm{Re}_{P,cr} = 3000$, and for Couette's flow $\mathrm{Re}_{C,cr} = 1300$ [1].

The described phenomenon of flows is a superposition of both these flows, the boundary conditions being described by two Reynolds numbers. Due to turbulization of the Couette's flow the coefficient of resistance of the flow through the clearance grows, whereas an increase of the Reynolds number in Poiseuille's flow is connected with a reduction of the coefficient of resistance [5, 6].

The flow in a face clearance can also be defined as an intermediate phenomenon between the flow in a longitudinal clearance with a rotating inner wall and the flow through such a clearance with a rotating external wall. In the first case the critical Reynolds number Re_C reaches a value of 1900, and in the latter case the flow becomes unstable already at the stage of laminar motion. This may be due to the effect of centrifugal forces during the curvilinear motion of the liquid, when the velocity from the center of the curve towards the external side of the flux in the former case grows and in the latter one decreases [15].

In the region of the inlet to the clearance its character is determined by Reynolds number, similarly as in the case of the flow around the rotating disc, and is defined by the formula [1]

$$\mathrm{Re}_M = \frac{\omega r^2}{\nu}. \tag{25}$$

Investigations quoted by Schlichting [1] indicate that if there is no forced flow, the flow around the rotating disc passes over from a laminar flow to a turbulent one when $\mathrm{Re} = 3 \cdot 10^5$. From Wagner's investigations [5] it results that in spite of a turbulent flow in front of the clearance, inside the clearance the flow becomes laminar. Thus, for instance, in front of the clearance $\mathrm{Re}_M = 4.47 \cdot 10^5$ but inside it we have a laminar flow: $\mathrm{Re}_P = 2300$, $\mathrm{Re}_C = 1300$. From the region outside the clearance the liquid flows between the disc and the casing with a considerable width into a very narrow clearance between the slip and the stopper rings. Such a change of the conditions of flow is connected with the qualitative increase of the force of internal friction versus the forces of inertia and laminarization. Only if $\mathrm{Re}_C = 1500$, the flow through the clearance changes its character more visibly.

In order to calculate the value of Re_P, the radial velocity in the clearance can be found using, for instance, the formula (13). Substituting $z = a/2$ we can calculate the maximum radial velocity, and the average velocity of the laminar flow can be defined as 2/3 of the maximum velocity. In the case of the discussed flow the average velocity in the clearance changes both along the radius and along the circumference.

If the width of the clearance varies, also the Reynolds number will change along the clearance as well as inside the clearance. In the zone of a narrower clearance there may occur a laminar flow, and in the zone with a greater width the flow can be turbulent.

8 Numerical example

The numerical values have been chosen in such a way that the flow would be laminar in the whole clearance, i.e. $\mathrm{Re}_P < \mathrm{Re}_{Pkr}$ and $\mathrm{Re}_C < \mathrm{Re}_{Ckr}$. For the sake of a better understanding of this description the following numerical values have been assumed: $\rho = 1000$ kg/m^3, $\mu = 0.0011404$ kg/ms (as for water at a temperature 15°C), $\omega = 150$ s^{-1}, $r_1 = 0.075$ m, $r_2 = 0.1$ m, $a_m = 0.00005$ m. Fundamental calculations were carried out for the angle of convergence of this clearance $\alpha = 0.00046$. The most possible convergence of the clearance will occur when $\varphi = \pi$ will be $a = a_{min} = 0$ amounting to $\alpha = 0.0005$ rad.

8.1 Centrifugal flow

At the inlet to the clearance the pressure on the radius r_1 amounts to $p_1 =$ 1200000 N/m^2, whereas the pressure at the outlet cross-section of the clearance it amounts on the radius r_2 to $p_2 = 600000$ N/m^2. Assuming these values, the pressure p along the circumference of the clearance and in expansion was presented on diagrams for the subsequent radii. Figure 2 shows the pressures calculated by means of Eq. (12), and Fig. 3 these same pressures calculated by means of equation (13b).

Comparing Fig. 2 and Fig. 3 we find that the solutions are quite similar and differ only numerically. In order to assess the differences in both cases the force F_a, the moments M_x, M_y, M, as well as the angles φ_K were calculated:

Calculations:	applying equation (12):	applying equation (13a):
	$F_a = 12394.4$ N	$= 12074.7$ N
	$\psi = 0.5029$	$= 0.4641$
	$M_x = 57.185$ Nm	$= 48.504$ Nm
	$M_y = 46.1$ Nm	$= 37.07$ Nm
	$M = 73.455$ Nm	$= 61.05$ Nm
	$\varphi_M = 0.6785$ rad $= 38.9°$	$= 0.6526$ rad $= 37.4°$

Disregarding motions of the order $< \varepsilon/(1+\varepsilon) = 0.333$ to 0.25 the differences between the calculated components M_x, M_y of the momentum M are of this order. Discrepancies between the values of the calculated forces F_a and angles φ_K, however, are much smaller. If the value of the angle φ_K is negative, the momentum acting on the slip-ring counteracts its contacting the stopper ring.

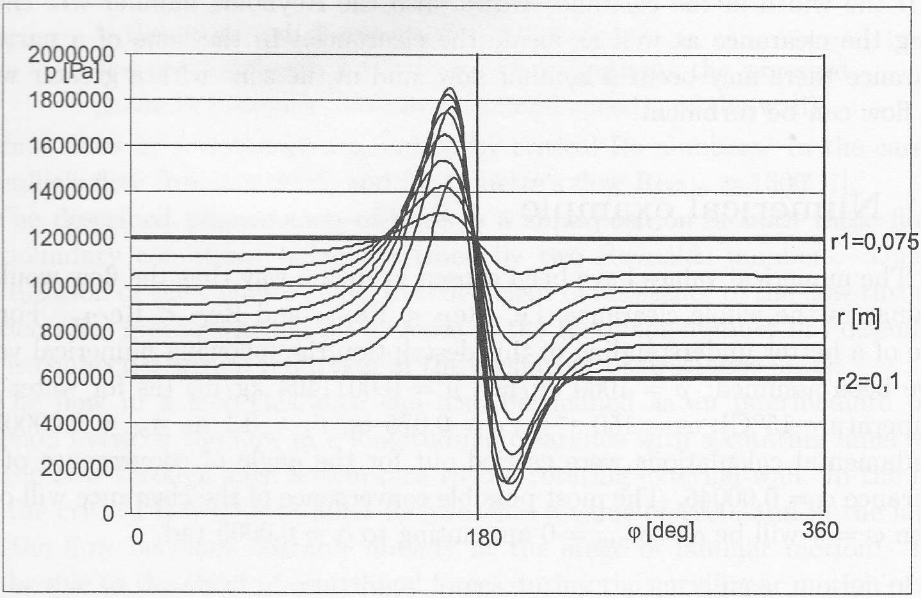

Figure 2. Centrifugal flow. Pressures field in the face clearance with a rotating slip-ring. Calculated by means of Eq. (12).

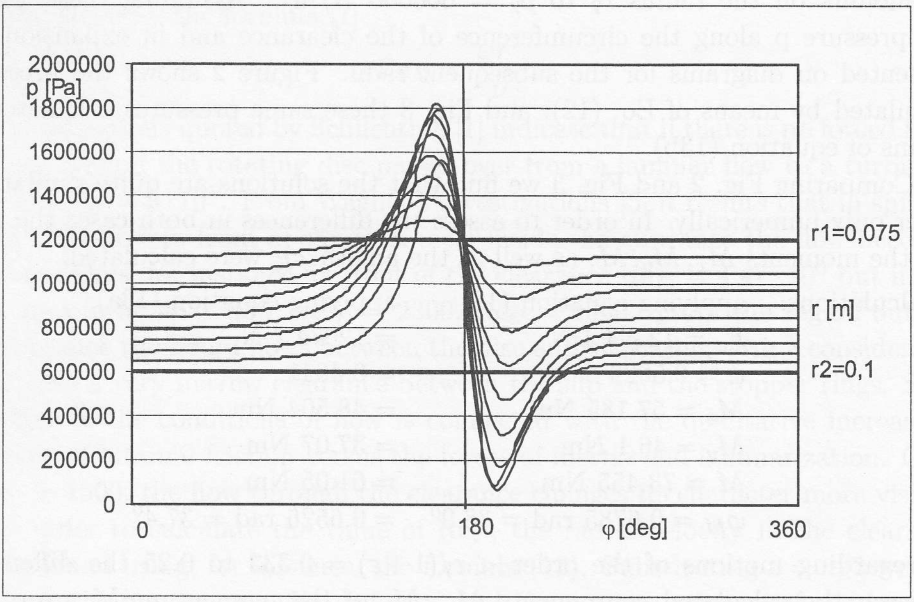

Figure 3. Centrifugal flow. Pressures field in the face clearance with a rotating slip-ring. Calculated by means of Eq. (13a).

Further results have been quoted for solutions obtained by applying Eq. (13a). Fig. 4 illustrates the influence of the convergence of the walls of the clearance on the maximum pressures along the radius, and Fig. 5 on the minimum pressures. From the diagrams presented in Figs. 4 and 5 it results, that with the growing convergence the pressure increases progressively in the clearance just before its narrowest part and drops immediately beyond it. Actually maximum pressures may be reduced due to the decrease of viscosity of the liquid, resulting from the rise of temperature brought about by friction. Also minimum pressures will be restricted to the boiling pressure of the liquid flowing through the clearance. Fig. 6 presents the dependence of constituent and complete moments on the convergence of the clearance. The intensity of the centrifugal flow, calculated by means of Eq. (16), amounts to $Q = 0.1226$ l/s. In the whole clearance the Reynolds numbers Re_P and Re_C are smaller than the critical ones. In the case of such an angle φ_M the moment M counteracts the convergence of the clearance.

8.2 Centripetal flow

Outside the clearance the pressure has been assumed to be $p_2 = 1200000$ N/m^2, and inside $p_1 = 600000$ N/m^2. Similarly as in the case of the centrifugal flow, the pressures were calculated along the circumference, as shown in Fig. 7. Then, analogically as in the case of the centrifugal flow, the force and the momentum were calculated, acting on the slip ring and the stopper ring at a centripetal flow. As a result of calculations the following values could be determined: $F_a = 11727$ N, $\psi = 0.42203$, $M_x = 48.504$ Nm, $M_y = -37.103$ Nm, $M = 61.07$ Nm, $\varphi_M = 0.653$ rad $= -37.410$, $Q = 0.1167$ l/s. In the case of such an angle φ_M the momentum M leads to an increase of convergence of the clearance (within the frame of the elasticity of the construction), i.e. the effect is opposite to that of the centripetal flow.

8.3 Flow in the case of immobile walls of the clearance

If the walls of the clearance are immobile, this phenomenon can be reduced to Poiseuille's flow through a flat-walled face clearance with a variable width. Then, in the case of a centrifugal flow the pressure is distributed in the clearance as shown in Fig. 8, whereas in the case of a centripetal flow the pressure is distributed as shown in Fig. 9. In the case of the centrifugal flow the calculated values were: $F_a = 12074$ N, $\psi = 0.4641$, $M_x = 0$ Nm, $M_y = M = 37.09$ Nm, $\varphi_M = (\pi/2)\mathrm{rad} = 90^\circ$. In the case the centripetal flow, on the other hand, these values amounted to: $F_a = 11726$ N, $\psi = 0.42203$, $M_x = 0$ Nm, $M_y = M = -37.09$ Nm, $\varphi_M = -(\pi/2)\mathrm{rad} = -90^\circ$. The diagram showing the distribution of pressures and the calculated values indicates that in the case of a centrifugal flow

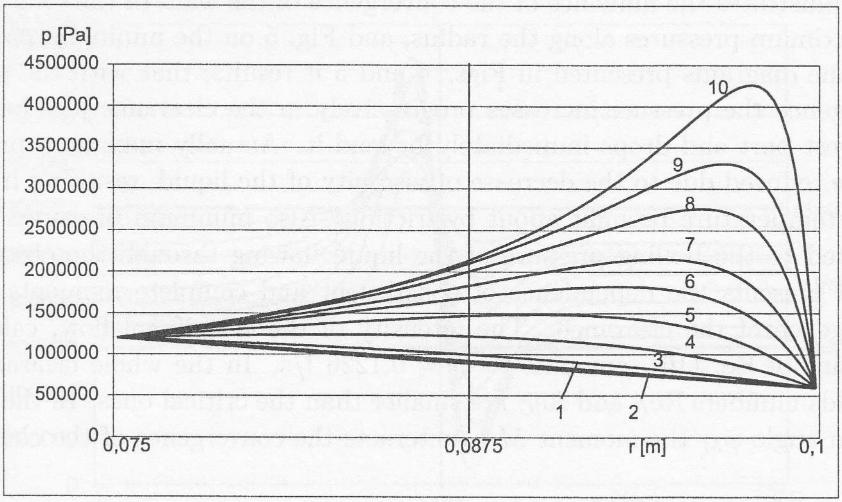

Figure 4. Centrifugal flow. Maximum pressures along the radius, at various convergences α of the wall of the clearance. α: 1)0.0001; 2) 0.0002; 3) 0.0003; 4) 0.0004; 5) 0.00044; 6) 0.00046; 7) 0.00047; 8) 0.00048; 9) 0.000485; 10) 0.00049.

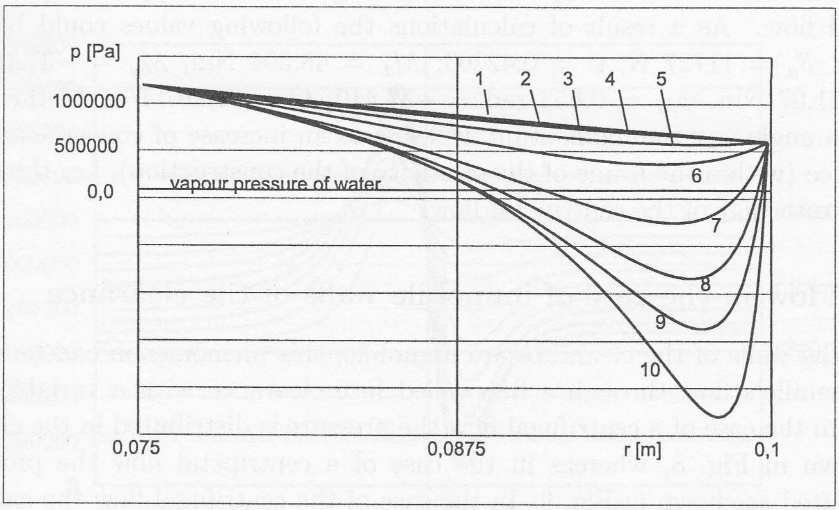

Figure 5. Centrifugal flow. Minimum pressures along the radius, at various convergences α of the wall of the clearance. α: 1)0.0001; 2) 0.0002; 3) 0.0003; 4) 0.0004; 5) 0.00044; 6) 0.00046; 7) 0.00047; 8) 0.00048; 9) 0.000485; 10) 0.00049.

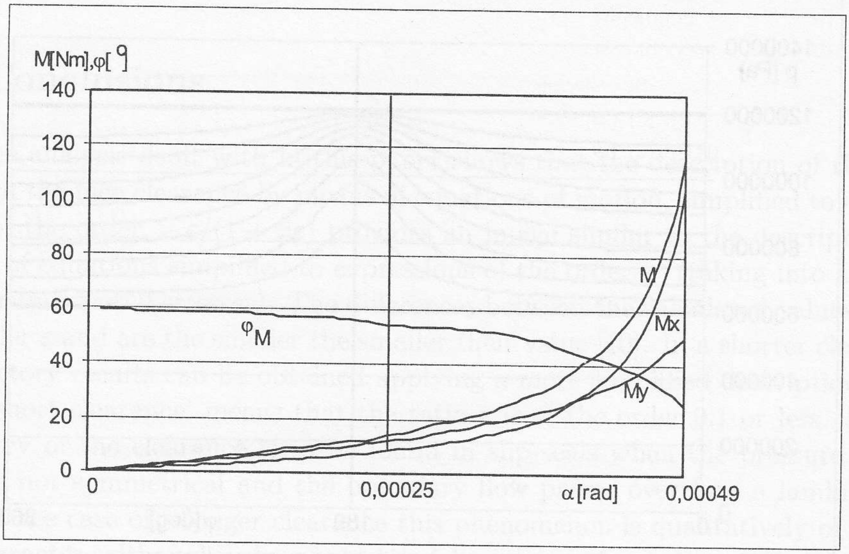

Figure 6. Dependence of constituent and full moments on the convergence α of the clearance and the angle φ_M between the vector of the momemntum M and the axis x.

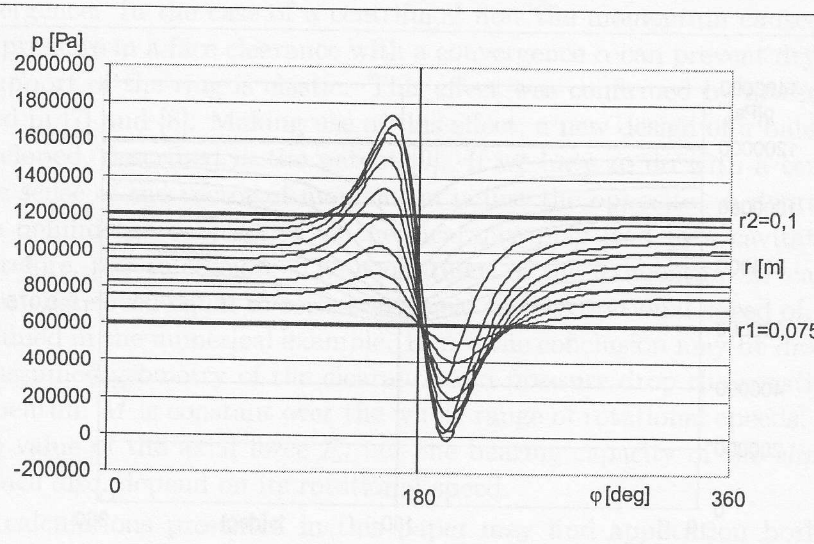

Figure 7. Centripetal flow. Pressure field in the face clearance with a rotating slip ring.

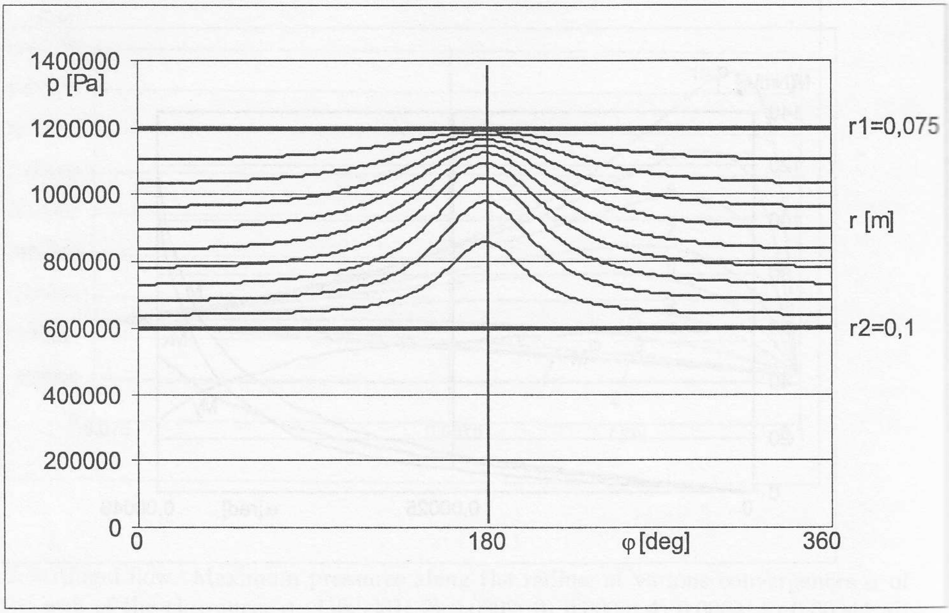

Figure 8. Centrifugal flow. Pressure field in the face clearance with immobile walls.

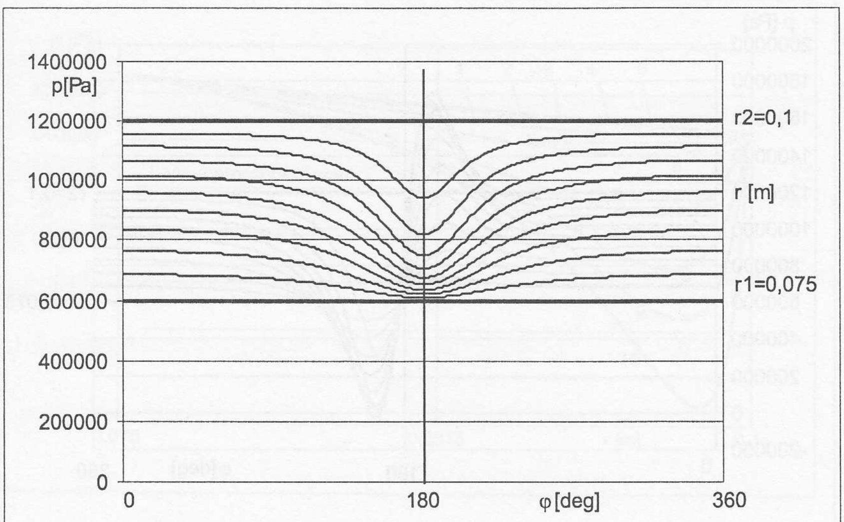

Figure 9. Centripetal flow. Pressure field in the face clearance with immobile walls.

the momentum counteracts the contacting of the walls of the clearance and that in the case of a centripetal flow just the opposite is the case.

9 Conclusions

The analysis dealt with in this paper shows that the description of the flow through the face clearance by means of equations of motion, simplified to expressions of the order $> \varepsilon/(1+\bar{x}\varepsilon)$ provides an image similar to the description by means of equations simplified to expressions of the order ε^3 (taking into account the circumferential stresses). The differences between the calculated values are of the order ε and are the smaller the smaller their value [10]. In a shorter clearance satisfactory results can be obtained applying a more simplified description. The term 'short clearance' means that the ratio ε is of the order 0.1 or less. Such a geometry of the clearance is to be found in slip-seals when the pressure of the rings is not symmetrical and the boundary flow passes over into a laminar one [10]. In the case of a longer clearance this phenomenon is qualitatively of a similar character, although values calculated by means of equation (12) may differ substantially from those calculated more precisely.

The presented analysis of a laminar flow through a face clearance indicates that the distribution of pressure inside the clearance depends to a large extent on its convergence. In the case of a centrifugal flow the momentum caused by the force of pressure in a face clearance with a convergence α can prevent dry friction if the support of the ring is elastic. This effect was confirmed by investigations described in [7] and [8]. Making use of this effect, a new design of a balance disc was developed, described in the patent [9]. If we have to do with a centripetal flow, the sense of the vector of momentum is just the opposite. A depression of pressure behind the contraction of the clearance may lead to a cavitation and give therefore, rise to erosion. The constituent of the momentum M calculated at a rotational speed equal to zero is identical to the rotational speed of, the *slip* ring assumed in the numerical example. Hence the conclusion may be drawn that for the assumed geometry of the clearance and pressure drop the constituent of the momentum M is constant over the whole range of rotational speeds. Neither does the value of the axial force F_a i.e. the bearing capacity of the *slip* ring of the balance disc, depend on its rotational speed.

The calculations presented in this paper may find application both in the suggested range of parameters of the operation of the balance disc in a multi-stage centrifugal pump and for axial slide bearings fed with oil under adequate pressure.

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