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ROMUALD RZĄDKOWSKI*

Comparison of the linear and nonlinear cumulative damage theories in transient analysis of a tuned and mistuned turbine blades

Institute of Fluid-Flow Machinery, PAS, Fiszera 14, 80-952 Gdańsk, Poland
Polish Naval Academy, Śmidowicza 71, 81-919 Gdynia, Poland

Abstract

Mechanical vibrations in turbine stages are one of the most significant reasons for a failure of bladed discs assemblies in turbines and compressors. A numerical example of a transient analysis during a run-up and a run-down process and the estimated fatigue life of the tuned and the mistuned bladed discs are presented. Life estimates are made using the linear and nonlinear cumulative damage theories and a comparison of the results is presented. It is done in order to show the importance of the consideration of the detuned system and the differences between the tuned bladed disc analysis.

The blade is modelled on a basis of extended beam theory including a bending-bending-torsional vibration. The disc is modelled by the moderately thick plate theory. The equation of motion is obtained by using the extended Hamiltons principle and the Ritz method.

Keywords: Linear and nonlinear cumulative; Tuned and mistuned turbine;

Nomenclature

C – damping matrix
D – dissipative force
E – elastic energy
$\mathbf{F}$ – vector of external forces
h – disc thickness
K – kinetic energy
$\mathbf{K}$ – stiffness matrix
N_e – limiting value of stress cycles endured
$\mathbf{M}$ – mass matrix
R – disc outer radius
S_e – endurance limit
S_y – yield stress
S_u – ultimate tensile stress
$\mathbf{u}$ – displacement of any particle of the bladed disc
$\mathbf{u}_d$ – displacement of any particle of the disc
$\mathbf{u}_i$ – displacement of any particle of i-th blade

*E-mail address: Z3@imp.gda.pl

$\mathbf{q}$	–	vector of generalized unknown displacements of the bladed disc	$\mathbf{U}$	–	matrix of displacement
r_0	–	disc inner radius	z	–	number of stator blades (number of nozzles)

1 Introduction

Mechanical vibrations in turbine stages are one of the most significant reasons for a failure of bladed discs assemblies in turbines and compressors. The dynamic response of the blade and bladed disc has been discussed in the literature by Rao and others [1-2], Irretier and others [3-4], Rieger and others [5-6] and Rządkowski[7-9]. These models allow calculation of the dynamic response of the bladed disc assemblies. Some of them even go further and perform life estimations of a single blade on the basis of appropriate theories (Rao and others [1-2], [10-11], Irretier and others [3-4], Rieger and others [5-6]). In the present paper the life is calculated for the mistuned and the tuned blade disc by using the known method. It is done in order to show the importance of the consideration of the detuned bladed disc.

2 Structural model

The blade model applied here is a one-dimensional beam described by an extended beam-theory including all important effects on a rotating blade. The beam is pre-twisted and tapered, with the variable cross-sectional area mounted at a stagger angle at the blade root. The model was described in detail in reference (see Rządkowski [7-8] and Janecki [12]). The disc is modelled by a moderately thick plate theory. The blades and disc are modelled as Hook's material. Assuming rigidly fixed blades on the disk rim, the displacements of any particle of the bladed disc is written in the form

$$\mathbf{u} = \mathbf{U}\mathbf{q}, \tag{1}$$

where: $\mathbf{u} = \text{col}\ (\mathbf{u}_1, \ldots, \mathbf{u}_N, \mathbf{u}_d)$, $\mathbf{q} = \text{col}\ (\mathbf{q}_1, \cdots, \mathbf{q}_N, \mathbf{q}_d)$, $\mathbf{u}_i$ is the vector of generalized displacements of each point of the ith blade, $\mathbf{u}_d$ is the vector of generalized displacements of each point of the disc, $\mathbf{q}_i$ are the unknown functions of the ith blade and the $\mathbf{q}_d$ are the unknown functions of the disc, both describing the displacement field. The matrices $\mathbf{u}, \mathbf{q}, \mathbf{U}$ are shown in (Rządkowski [7-8]).

In order to find an approximate solution of the forced vibrations of the mistuned bladed disc, the Hamilton principle can be used

$$\delta \int_{t1}^{t2} (E - K)dt = \int_{t1}^{t2} \delta W - D)dt, \tag{2}$$

where: E and K are elastic and kinetic energies, respectively and δW and δD are the variations of external work and the internal dissipative forces, respectively.

Following the above equations, the equation of motion of the bladed disc for the forced vibration can be found.

3 Stress results

The transient analysis during a run-up and a run-down process and the estimated fatigue life of the tuned, the mistuned bladed discs and a single free standing blade are presented.

The bladed disc considered here is a compressor bladed disc of a gas turbine with 30 blades (see [4]). The important properties of the bladed disc are as given below: the disk inner radius: $r_0 = 0.05$ m, the bladed-disk junction radius: $R = 0.316$ m, the blade length: $L = 0.228$ m. The material stress data of the blade made of steel X 10 Cr 13 for an operating temperature of 300 ^{o}C are taken as: the ultimate tensile strength $S_u = 610$ MPa, endurance limit $S_e = 275$ MPa, yield stress $S_y = 495$ MPa, and the Young Modulus 216 GPa. The density of the material is taken as 0,00785 kg/cm^3. All geometrical parameters of the blade are presented in [7]. The Campbell diagram giving resonant rotor speeds is presented in Fig. 1, in order to comment on the transient response of the system.

In the present study an operation is simulated where the bladed disc runs up from a standstill to a rotational speed of 3000 rpm within 25 s, and next runs down to the standstill within the 25 s. During the simulated process the blade was subjected to the mean values of lift force $L_0 = 1$ N/mm and drag force $D_0 = 0.1$ N/mm and additionally the first harmonic of excitation in a stage of $z = 30$ nozzles of the unsteady lift $L_1/L_0 = 0.02$ and drag $D_1/D_0 = 0.002$. The excitation amplitude is 2% of the mean value of lift and 0.2% of the drag respectively. The natural frequencies of tuned and mistuned blade discs could be classified into groups. Each group consists of N frequencies, where N is the number of blades, [7]. The viscous damping ratios for the first group is $\xi_1 = 0.00182$, for the second $\xi_2 = 0.00126$ and third $\xi_3 = 0.00051$, respectively, [10]. Several different types of blades could be considered by using the coefficients μ_i, [9]. For example, the cross-sectional area of the detuned blade can be calculated from equation $A_i = \mu_i^2 A$, where A is a cross-sectional area of the tuned blade. This type of mistuning does not change the length of the blade, only the cross-sectional area.

The response of the bladed disc during run-up, for different arrangements of the detuned blades around the disc, was analysed in [7] and [9]. It was found that the random detuning caused the maximum stress increase in comparison to that of the tuned state.

In this paper only one detuned system is considered, where the blades differ from each other in the range (–1, 1)%. Three kinds of blades ($\mu_1 = 1, \mu_3 = 0.99, \mu_2 = 1.01$) are arranged at random (3, 1, 1, 1, 3, 1, 2, 2, 2, 1, 2, 3, 3, 1, 2, 1, 3, 3, 2, 1, 3, 1, 2, 1, 3, 3, 2, 1, 3, 1) (see Fig. 2).

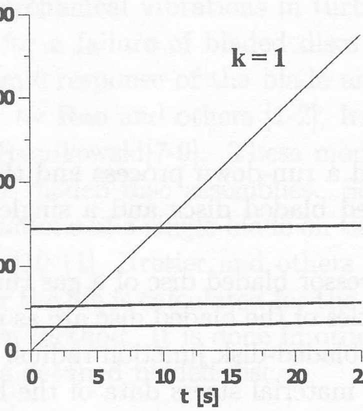

Figure 1. Campbell diagram of the bladed disc for a zero nodal diameter.

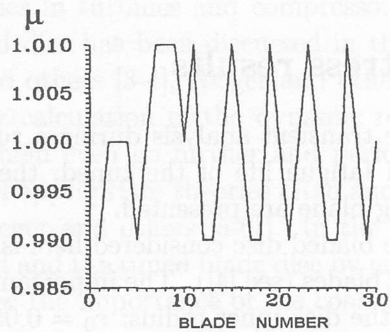

Figure 2. Arrangement of detuned blades around the disc.

The normal stresses τ_{33} of the tuned bladed disc and the mistuned bladed disc during run-up and run-down are calculated using the modal superposition method. The algorithm and mathematical formulas are presented in details in [7] and [9]. In this case, the first 60 mode shapes of the bladed discs are taken into consideration in the modal superposition analysis, [13, 7].

It is known that the bladed disc possesses a spectrum of eignfrequencies which generally depend on the angular speed. At each point of intersection of the eigenfrequency curve with one of the excitation lines in the Campbell diagram (see Fig. 1), the resonance vibration occurs. Figures 3-6 present the maximum blade stress τ_{33}, as a function of time, of the mistuned system and the tuned bladed discs, during run-up and run-down, respectively. The response picture of the mistuned bladed disc (Figs. 3-4) is slightly different from the response picture of the tuned bladed disc (Figs. 5-6). The number of resonances is higher in the mistuned case. It follows from the fact that for the mistuned system the number of natural frequencies is also higher, [7] and [9].

Another interesting comment on the response of that particular blade is, that the maximum stress appeared very suddenly (for example see Fig. 3 for the time $t \approx 4$ s), and next slowly continues to decrease. For another blades, (see [4]), the

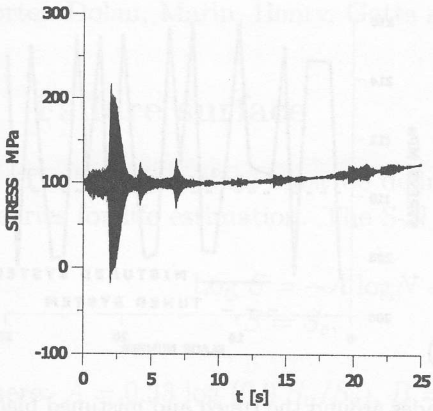

Figure 3. The transient blade response of the mistuned bladed disc with 30 blades during the run-up.

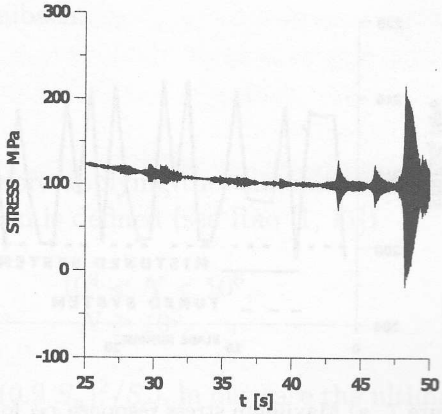

Figure 4. Transient blade response of the mistuned bladed disc with 30 blades during the run-down.

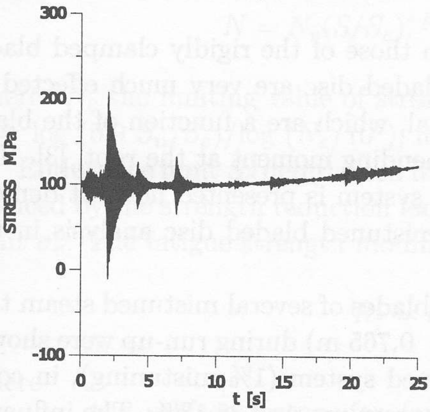

Figure 5. Transient blade response of the blade of the tuned bladed disc during the run-up.

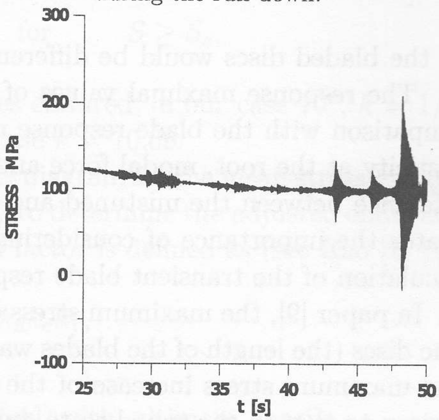

Figure 6. Transient blade response of the blade of the tuned bladed disc during the run-down.

response around the resonances can be much more symmetrical. The maximum stress distribution of the 30 blades for a considered mistuning is presented in Figs. 7a and 7b. All the curves show the mean stress as the sum of the static bending stress of the mean values of the lift and drag forces and the normal stress due to the centrifugal force field, which increases during the run-up phase of the operation up to its final constant value while it decreases during the run-down period correspondingly.

The comparison of the response picture between the rigid clamped single blade and tuned bladed disc during run-up was shown by Irretier and Omprakash [3]. The natural frequency spectrum of the bladed disc is considerably influenced by the blade-disc elastic coupling and hence the points of resonance

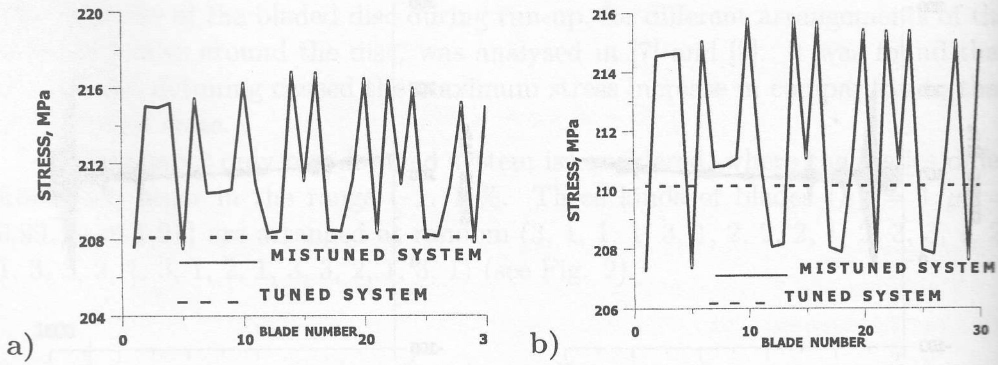

Figure 7. a) Maximum stress response τ_{33} for all blades around the tuned and mistuned bladed disc with 30 blades during run-up; b) Maximum stress response τ_{33} for all blades around the tuned and mistuned bladed disc with 30 blades during run-down.

for the bladed discs would be different from those of the rigidly clamped blade, [3]. The response maximal values of the bladed disc are very much effected in comparison with the blade response maximal, which are a function of the blade elasticity at the root, modal force and the bending moment at the root, [3]. The difference between the mistuned and tuned system is presented here. It demonstrates the importance of considering the mistuned bladed disc analysis in the calculation of the transient blade response.

In paper [9], the maximum stress of the blades of several mistuned steam turbine discs (the length of the blades was $L = 0.765$ m) during run-up were shown. The maximum stress increase of the mistuned system (1% mistuning), in comparison to that in the tuned state during the run-up, was 35.13%. The influence of mistuning of the compressor bladed disc with the shorter blades $L = 0.228$ m is presented here. Figs. 7a, 7b present the maximum stress τ_{33} at a particular blade around the tuned system and mistuned bladed disc during the run-up and run-down respectively. The maximum stress increase of the mistuned system (1% mistuning) in comparison to that in the tuned state during the run-up was 10.42% and during the run-down was 9.7%. Having the maximum dynamic response curves versus time a classification procedure is required to convert this function into a stress level distribution. There exist various techniques dealing with this problem, [17]. Here, the rainflow-cycle counting technique are realized in the life estimation procedure.

The influence of mean stress is taken into account by the fourth order relation of Bagci [14], which is found to fit well experimental data for steels. When the alternating stress is of variable amplitude, several cumulative damage theories are proposed, the most common of them being the linear one due to Palmgren and Miner. The nonlinear theories proposed include these due to Marko-Starkey, Corten-Dolan, Marin, Henry, Gatts and Haibach.

4 Failure surface

The failure surface must be defined before applying the cumulative damage theories for life estimation. The S-N diagram is defined (see Rao [1, 11])

$$\begin{aligned} \text{Log}\, S &= -A \log N + B, & 10^3 < N < 10^6, \\ S &= S_e, & N \geq 10^6, \end{aligned} \tag{3}$$

where: $A = 0.33 \log\,(0.9\, S_u/S_e)$, $B = \log\,[(0.9\, S_u)^2/S_e]$, in our case the ultimate tensile strength $S_u = 610$ MPa, the endurance limit $S_e = 275$ MPa. Equation (3) can be written in the form

$$N = N_e(S/S_e)^{-k}, \qquad \text{for} \qquad S \geq S_e,$$

where: N_e the limiting value of stress cycles endured, in our case 10^6, $k = 1/A$, $A = \log\,(0.9\, S_u/S_e)/\log\,(N_e/10^3)$, in our case $k = 10.09$.

Endurance limit S_e determined from S-N diagram for the mean stress $\sigma = 0$, is reduced by the strength reduction factor R_f to determine the adjusted endurance limit σ_e. The fatigue strength modification factor is defined as (see Rao [1, 11])

$$R_f = k_a k_b k_c k_d k_e k_f, \tag{4}$$

where

- k_a – the surface factor, for forged steel with ultimate tensile strength equal to 610 MPa, $k_a = 0.45$;
- k_b – thesize effect factor (see Rao [11]), $k_b = 1$ for $d < 10$ mm, where d is the effective thickness;
- k_c – the reliability factor k_c corresponding to 8% standard deviation of the endurance limit for 95% reliability is 0.868 (see Rao [11]);
- k_d – the temperature factor $k_d = 1$ for $T \leq 450$ °C;
- k_e – the stress concentration factor, $k_e = 1$;
- k_f – the miscellaneous factor, $k_f = 1$.

The fatigue strength modification factor R_f works out to be 0.3906. So the following has been obtained

$$\sigma_e = R_f S_e.$$

When the mean stress at the point of interest does not vanish, the effect of the mean stress on the fatigue strength reduction is considered by employing fatigue failure criterion defining the adjusted fatigue surface for the specific number of stress cycles to fatigue. In our case the fourth order relation of Bagci is used

$$\sigma_{af} = \sigma_e[1 - (\sigma_m/S_y)^4],$$

where: S_y is the yield strength, σ_m is the mean stress value, σ_{af} is the alternating stress. So S-N diagram is defined

$$\begin{array}{ll} \text{Log}\, S = -A \log N + B, & 10^3 < N < 10^6, \\ S = \sigma_e[1 - (\sigma_m/S_y)^4], & N \geq 10^6, \end{array} \tag{5}$$

where: $A = 0.33 \log\,(0.9\ R_f S_u / R_f S_e)$, [14],
$B = \log\{(0.9\ R_f S_u)\ [1 - (\sigma_m/S_y)^4]/0.9\ R_f S_e\}$, so $k = 10.09$.
Equation (5) for i-th amplitude σ_{ai} can be written in the form (classical Wöehler-line)

$$\begin{array}{lll} N_i = N_e(\sigma_{ai}/\sigma_{afi})^{-k} & \text{for} & \sigma_{ai} \geq \sigma_{afi}, \\ \infty & \text{for} & \sigma_{ai} < \sigma_{afi} \end{array}. \tag{6}$$

5 Cumulative damage

5.1 Palmgren-Miner linear damage theory, [15], [16]

Consider the S-N diagram of a given machine part. Operation at a stress level S_i gives a life of N_i cycles. If the blade is subjected to n_i cycles, it suffers a damage fraction $D_i = n_i/N_i$. Failure is then predicted to take place when

$$D = \sum D_i = n_1/N_1 + \ldots n_i/N_i \geq 1. \tag{7}$$

It asserts that the damage fraction at any stress level is linearly proportional to the ratio of number of cycles of operation to the total number of cycles that would produce failure at that stress level. The value N_i can be found having the amplitude σ_{afi} from $\sigma_a - N$ diagram in form of Eq. (6). The stress amplitudes below the fatigue strength do not contribute to the estimated fatigue damage. This assumption, however, is known to be insufficient for the damage process of stochastic stress in many cases. Modified hypotheses of cumulative fatigue damage were developed which also consider the damage contribution of these stresses. In particular, the hypothesis of Haibach [17] and that described by Corten and Dolan [18] can be considered (see Tab. 1).

Haibach [17] suggests the usage of a double line S-N diagram, the first one similar to the classical S-N line and the second one that continues into region of fatigue strength with a different slope

$$\begin{array}{ll} N_i = N_e(\sigma_{ai}/\sigma_{afi})^{-k} & \sigma_{ai} \geq \sigma_{afi}, \\ (8) N_i = N_e(\sigma_{ai}/\sigma_{afi})^{-(2k-1)} & \sigma_{ai} < \sigma_{afi} \end{array}, \tag{8}$$

which corresponds to the continuation of the decreasing curve of stress amplitude versus the number of cycles, also beyond the limiting value of stress cycles endured by a straight line with approximately half of the slope than above this point.

Corten and Dolan suggest a modification of S-N diagram to be defined by one line in the entire region given by

$$N_i = N_e(\sigma_{ai}/\sigma_{afi})^{-k^*}, \tag{9}$$

where: N_e is the limiting value of stress cycles endured, k^* lies between 0.8 k and 0.9 k (taken as 0.85 k in the present case) and k is an exponent depending on the material, which yields stress-cycles with slope more than the classical S-N diagram, but continues into the range of fatigue strength.

One of the serious drawback of the linear theory is that it does not recognise the order of application of various stress levels and damage is assumed to accumulate at the same rate at a given stress level without consideration of history.

Table 1. Life Estimates (cycles x 10^{-7}) for a single blade of bladed tuned disc during run-up

Classical S-N	Coten-Dolan	Haibach
2.35	9.28	5.46

5.2 Marco-Starkey damage theory, [19]

Marco and Starkey suggested that the damage for each level of reversed sinusoidal stress amplitude may be represented by the exponential relationship

$$D_i = (n_i/N_i)^m, \tag{10}$$

rather than the linear equation assumed by Miner, where the exponent m depends on the amplitude of the alternating stress. One of the problems with the application of Marco-Starkey's theory is the lack of material data for the exponent m for different values of stress levels. A study of sequential load test data for SAE 4320 steel obtained by Marco-Starkey given in Tab. 2 (Rao at el. [10]) suggests a simple procedure to account for the non linearity in the damage.

The value of $\sum(n_i/N_i)_{avg}$ gives the average for all the specimens tested under a particular set of values of the first stress level σ_1 and the last stress level σ_2. The value of $\sum(n_i/N_i)_{\max}$ was computed for one specimen of the set which showed the longest total life. The value $\sum(n_i/N_i)_{min}$ was computed for one specimen of the set which showed the shortest total life. When the stress level is increased from 550 MPa to 825 MPa in the number of steps $N = 20$ in a continuously increasing

Table 2. Sequential load test results for SAE 4340 steel [19], [11]

First stress level	Last stress level	No. of steep	Avg. $\sum(n_i/N_i)$	Max. $\sum(n_i/N_i)$	Min. $\sum(n_i/N_i)$	N
550	825	20	1.05	1.4	0.9	6
825	550	20	0.6	0.79	0.57	13

manner, the average values of $\sum(n_i/N_i)$ is equal to 1.05, the maximum value of $\sum(n_i/N_i)$ is equal to 1.4, the minimum value of $\sum(n_i/N_i)$ is equal to 0.9. Similarly, when the stress level is decreased from 825 to 550 MPa in the number of steps, the mean value of $\sum(n_i/N_i)$ is equal to 0.6, the maximum value of $\sum(n_i/N_i)$ is equal to 0.79, the minimum value of $\sum(n_i/N_i)$ is equal to 0.57. For the period when the stress levels are increasing to peak amplitude, the mean value of $\sum(n_i/N_i)$ in this period may be multiplied by a factor $f = 1/1.05$, the maximum value of $\sum(n_i/N_i)$ in this period may be multiplied by a factor $f = 1/1.4$, the minimum value of $\sum(n_i/N_i)$ in this period may be multiplied by a factor $f = 1/0.9$. For the period when the stress levels are decreasing from the peak amplitude, the mean value of $\sum(n_i/N_i)$ in this period may be multiplied by a factor $f = 1/0.6$, the maximum value of $\sum(n_i/N_i)$ in this period may be multiplied by a factor $f = 1/0.79$, the minimum value of $\sum(n_i/N_i)$ in this period may be multiplied by a factor $f = 1/0.57$.

In our case the stress response around the resonance was discretized to at least 20 steps in either side of resonance. The results of the life for a tuned bladed disc during run-up are presented in Tab. 3.

Table 3. Life estimates (cycles x 10^{-7}) for a blade of the tuned bladed disc during run-up

Palmgren-Miner	Marco-Max	Marco-Avg	Marco-Min
2.35	2.98	3.93	4.14

5.3 Corten-Dolan Damage theory, [18]

This theory recognises the need for postulating the initiation of permanent damage, somewhat similar to fracture mechanics theory, bringing it closer to the advanced concepts of fatigue failures, see Barsom and Rolf [20]. Using this property, they have shown that failure can be expected when

$$D = n_1/N_{\max} + n_2/N_{\max}(\sigma_2/\sigma_{\max})^d + \ldots + n_l/N_{\max}(\sigma_l/\sigma_{\max})^d, \quad (11)$$

where $N_{\max}$ is the number of cycles to failure at the highest stress amplitude $\sigma_{\max}$ and n_l are the total number of cycles imposed at each stress level σ_l. For steel the exponent d is found to be in the range between 6.2 and 6.9 with the mean value 6.57.

Corten and Dolan suggest a modification of S-N diagram to be defined by one line in the entire region see equation (9).

5.4 Marin's theory, [21]

Marin assumed that equivalent number of cycles n_1 at reference level, say S_1, that would produce the same damage as n_i cycles of operation at S_i level of stress, can be expressed as

$$n_{ie} = n_1(S_i/S_1)^y, \tag{12}$$

where y is an exponent that can be determined experimentally.
The damage ratio

$$D = n_1/N_1 + n_2/N_2(\sigma_2/\sigma_{\max})^q + \ldots + n_l/N_l(\sigma_l/\sigma_{\max})^q, \tag{13}$$

where $q = k - 0.657\text{k} = 0.343\,k$.

5.5 Henry's theory, [22]

When referring to the S-N curve the number of cycles N, of life expected at a given stress level S, is called the "endurance life" at that stress. "Stress" or "Stress level" refers to the amplitude of a stress variation which is sinusoidal. "Overstress" is defined as the ratio of the amount by which the stress exceeds the endurance limit to the endurance limit. In the case of steel, overstressing manifested itself in one of two general ways in so far as its effects on subsequent fatigue behaviour are concerned. It reduces the endurance life at all stress levels which are sufficiently high to produce failure. It affects the endurance limit, thus affecting the ability of the specimen to withstand subsequent stressing.

Henry accounted for the fact that the fatigue limit gets reduced when a specimen suffers damage, i.e., the S-N curve gets shifted as damage accumulates. The relation between the damage D and the fatigue limit is expressed as

$$D = (S_e - S_{ed})/S_e, \tag{14}$$

where S_e is the original fatigue limit of the virgin material and S_{ed} is the fatigue limit after the damage reduced from S_e.

The S-N curve itself is assumed to be represented by an equilateral hyperbola refereed to the stress axis S, and a line passing through S_e parallel to the N axis as the asymptotes.

The above theory due to Henry can be extended to a sequence of different alternating stress amplitudes in the order of applied stress levels. Each time of application of a stress level $\sigma_{ai} > \sigma_{afci}$, the current fatigue limit σ_{afci} should be updated. Thus, a sequence of the updated fatigue limits will be obtained, say $\sigma_{afc1}, \sigma_{afc2}, \ldots$ after application of n_1 cycles of stress level σ_{a1}, then n_2 cycles of stress level σ_{a2} and so on. Henry's theory therefore allows us to determine the diminishing fatigue limit as damage accumulates. Thus

$$D = \sum n_i/(N_i A_i), \tag{15}$$

where

$$A_i = 1 + [\sigma_{afci}/(\sigma_{ai} - \sigma_{afci})](1 - n_i/N_i) \quad \text{for} \quad \sigma_{ai} > \sigma_{afci}, \tag{16}$$

$$\sigma_{afci} = \{\sigma_{ai}(1 - n_i/N_i)\}/\{(\sigma_{ai} - \sigma_{afi})/\sigma_{afi} + (1 - n_i/N_i)\}. \tag{17}$$

5.6 Gatts' cumulative damage theory, [23]

Gatts' damage theory is based on the dependence of fatigue strength and fatigue limits on the number of cycles of stress and that such change is proportional to a damage function D(S), i.e.

$$DS_i/dn = k_1 D(S), \tag{18}$$

where: S_i is the instantaneous value of strength, n is the number of stress cycles applied, k_1 is constant of proportionality and D(S) is the damage expressed as a function of stress level.

Failure is then predicted to take place when

$$D = \sum n_i/N_i = \sum A_i/B_i, \tag{19}$$

where

$$A_i = 1/(\sigma_{ai} - \sigma_{afi}) - 1/(\sigma_{ai} - \sigma_{afci}), \tag{20}$$

$$B_i = (\sigma_{ai} - \sigma_{afi}) - 1/\{\sigma_{ai}(1 - C)\}, \tag{21}$$

$$\sigma_{afci} = \sigma_{ai}[1 - 1/\{(n_i/N_i)/C + \sigma_{ai}/(\sigma_{ai} - \sigma_{afi})(1 - n_i/N_i)\}. \tag{22}$$

The material constant C is usually found to be 0.5, for most of the steels. This equation provides a direct method for calculating the effect on the endurance limit of fatigue damage specified in terms of cycle ratio and stress-amplitude ratio. This equation is similar to the equation developed by Henry from consideration of the difference in the S-N diagram for damaged specimens as compared to the S-N diagram for undamaged specimens, [22]. For $C = 0$, the above reduces to σ_{afci} from the Henry's theory.

5.7 Manson's double linear damage theory, [24]

In this theory, the concepts of fracture mechanics are implied by recognising crack initiation and propagation aspects during the damage suffered by a specimen. The crack initiation period is denoted N' and the crack propagation period is defined by the number of cycles for failure after the crack has been initiated and denoted by $N_P = PN_j^p$ then

$$N' = N_f - N_P = N_f - PN_f^P, \tag{23}$$

where N_f is the total number of cycles for failure including the crack initiation, P is propagation coefficient and p is propagation exponent.

Manson found $p = 0.6$ and $P = 14$ from several experimental results, thus

$$N_P = 14N_j^{0.6}, \qquad N' = N_f - 14N_f^{0.6}. \tag{24}$$

The above results are found to be fairly accurate when $N_f > 730$ cycles. For failure occurring with less than 730 cycles, it is found that the crack initiation takes place in the first cycle of loading, because of the high stress level that is involved with very short lives. Miner's rule is then adopted by Manson for the crack initiation and propagation phases separately. Fatigue nuclei of critical size are initiated when

$$n_1/N_1' +, \ldots, +n_m/N_m' = 1. \tag{25}$$

This equation is applied until a crack is initiated. Fatigue cracks are then propagated to failure after cracks of critical size have been initiated and when

$$n_1/N_{P1}' +, \ldots, +n_q/N_{Pq}' = 1. \tag{26}$$

In both of the above phases, n is the number of cycles applied at the ith or jth stress level. The above equation then determines the life for crack propagation period.

6 Numerical results

The life estimation of tuned and mistuned bladed disc during run-up and rundown obtained by different cumulative damage theories is compared in Tabs. 3-5.

Marco-Starkey theory predicted lower life in comparison to the linear theory. The effect of decreasing stress amplitude is predominant. Marin's and Corten-Dolan's theories predict also higher life over linear theory and Manson's theories. Manson's theory estimated life similar to Palmgren-Miner values.

Henry's and Gatts' theories give the wrong results here. The blade has accumulated a very small amount of fatigue damage for a high stress, because of the low value of n_i maximum stress cycles, so in Eq. (15) values $A_i = 1 + [\sigma_{afci}/(\sigma_{ai} - \sigma_{afci})](1 - n_i/N_i)$ assumed the values bigger then 2 so the results of the life is smaller in comparison to the other theories. The similar situation can be observer for the Gatts' theory.

Table 4. Life estimates (cycles x 10^{-7}) for a tuned bladed disc during run-up

Palmgren-Miner	Marco-Starkey	Corten-Dolan	Marin	Henry	Gatts	Manson
2.358	3.930	9.280	6.796	0.04	0.04	2.415

Life estimates (cycles x 10^{-7}) for a tuned bladed disc during run-down

Palmgren-Miner	Marco-Starkey	Corten-Dolan	Marin	Henry	Gatts	Manson
5.502	9.171	14.947	8.989	0.172	0.171	5.645

Table 5. Life estimates (cycles x 10^{-6}) for a mistuned bladed disc during run-up

Palmgren-Miner	Marco-Starkey	Corten-Dolan	Marin	Henry	Gatts	Manson
1.191	1.984	3.356	2.426	0.08	0.08	1.227

Life estimates (cycles x 10^{-6}) for a mistuned bladed during run-down

Palmgren-Miner	Marco-Starkey	Corten-Dolan	Marin	Henry	Gatts	Manson
1.114	1.856	3.065	2.186	0.06	0.06	1.146

7 Conclusions

The results presented here (see Tabs. 4-5) emphasise importance of the influence of detuning and the elastic coupling of the blade-disc on the expected fatigue life. The analysis of the single blade and the tuned bladed disc are not sufficient.

Marco-Starkey theory predicted lower life in comparison to the linear theory. The effect of decreasing stress amplitude is predominant. Marin's and Corten-Dolan's theories predict also higher life over linear theory and Manson's theories. Manson's theory estimated life similar to Palmgren-Miner values.

Henry's and Gatts' theories give wrong results here.

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