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Determination of the amount of medium in working chambers of multi-vane rotational machines

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Abstract

The paper presents a comparison of flow types, which occur in rotational and volumetric machines. Leak paths in multi-vane rotational machines are indicated and their characteristics, i.e. length, width and height of their associated gaps, given. Formulae are also presented, which allow to calculate the individual gas mass fluxes related to the gaps. Based on the formulae, the gas mass contained in the working chamber is given for an arbitrary chamber position. This gas amount fluctuates strongly when the chamber is being filled or evacuated, although, in theory, it should remain constant in a closed chamber. As can be seen from the diagram that recapitulates the calculations by the author, the gas mass present in the working chamber during decompression decreases down to even 70% of its initial mass. Therefore, laws of the variable-mass thermodynamics have to be used to analyze thermodynamic processes performed by the gas, which is contained in the working chamber of a multi-vane rotational machine.

Keywords: Multi-vane rotational machine; Gas flow, Leakage

Nomenclature

$A, A(\varphi)$	–	area; area of the figure whose position is described by angle φ, m^2	h	–	vane height, m
B	–	vane width, m	k, n	–	exponent of adiabatic/polytropic curve
b_s	–	gap width, m	K_p	–	flow ratio
d_{Hs}	–	hydraulic radius of gap, m	l_s	–	gap length, m
e	–	eccentricity, m	L	–	vane length, m
			m, m_w	–	mass; mass of gas in the chamber at the moment it is being closed, kg

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$\overline{m}$ – relative mass of gas in the chamber
$\dot{m}_S, \dot{m}_{Sj}$ – gas mass flux, kg/s; gas mass flux related to 1 m^2 of gap, kg/(sm^2)
n_{ob} – shaft rotations per second, s^{-1}
$p, p_k, p_0, p_{SS}, p_t, p_w, p_z$ – absolute pressure; final, ambient, suction, forcing, inlet and tank pressure, respectively, Pa
$p(\varphi)$ – pressure in the chamber, whose position is described by angle φ, Pa
r, R – rotor radius; cylinder radius, m
R_i – individual gas constant, J/kg K
Re – Reynolds number
s, s_l, s_r, s_w, y – gap height; height of the vane's lateral gap; height of the vane's front gap; height of the rotor's front gap; height of radial gap, m
$s(x)$ – gap height for coordinate x, m
$T, T_w, T_z, T(\varphi)$ – gas temperature; gas temper. at inlet, in the tank and in this working chamber, whose position is described by angle φ, respectively, K
V_z – tank volume, m^3
w, w_{sc} – velocity; velocity of side wall, m/s
$x(\varphi)$ – length of this (projecting from the groove) vane, whose position is described by angle φ, m
z – coordinate to describe channel height, m
z_j – number to vanes
z_s – coefficient of compressibility
$Z(\varphi), Z(\varphi + \Delta\varphi_1)$ $Z[\varphi(\varphi_s = \alpha_1)], \dots$ – relative cross-section area of this working chamber, whose position is described by angle $\varphi, \varphi + \Delta\varphi_1, \varphi(\varphi_s = \alpha_1), \dots$; this area value is obtained by multiplying the relative cross-section area by R^2 [1]
$\alpha_1 \div \alpha_4$ – angles that describe those positions of the edges that controll the operation of a multi-vane machine, rad
β_{kr} – critical pressure ratio
$\Delta\varphi_i$ – i-th increase of angle φ, during which leveling up of chamber pressure occurs, rad
ν – coefficient of dynamic viscosity, Pa·s
λ – angle between two consecutive vanes, rad
$\rho_s(\varphi)$ radius vector of the contact point between the cylinder and vane, whose position is described by angle φ, m
φ – coordinate of polar coordinate system, rad
φ_s – coordinate that describes position of vane-cylinder contact point, rad
ω – rotational velocity, rad/s

1 Introduction

Although rotational vane machines belong to the category of volumetric fluid-flow power machines, the flow processes inherent to them are (at least partially) different from this type of medium flow that occurs in rotodynamic machines. Some of these differences are listed in the table below.

Rotodynamic machines	Volumetric machines
1. Basic form of substance transport between the inlet and outlet stud.	
• continuous flow	• batch flow (quantified)
2. Flow phenomena accompanying the basic form of transport	
• flow through diffusors and nozzles	• filling a constant-volume or variable-volume chamber

• flow through a blade cascade • external leaks (through sealings) • internal leaks (through radial and axial clearances) • flow within the boundary layer • shock wave • turbulence	• evacuating a constant-volume or variable-volume chamber • movement of gas contained in a chamber with variable volume • external leaks (through sealings) • internal leaks (including those from the inlet area that bypass the working chamber, between adjoining chambers), • flow within the boundary layer
3. Working medium velocity	
• high and very high, often approaching or exceeding the speed of sound	• predominantly low or medium (less than 100 m/s), rarely high
4. Stability of flow phenomena at an arbitrary point of flow channel	
• steady flow, or the one whose characteristic varies almost monotonically during machine startup or load variation; moderate rate of parameter variation	• flow with a cyclically changing characteristic; very high rate of changes (up to a few hundred per second); steady flow is obtained when the parameters that describe it change identically in consecutive cycles.

The differences listed above show clearly that separate treatment of flow phenomena in vane rotational machines is well grounded.

The transport of working medium from the inlet area to the outlet area, which occurs in the working chamber, is inevitably accompanied by the flow through gaps. Its effect on the operation of vane machines is by no means simple. A part of the medium passes directly from the high-pressure area (e.g. from the inlet area of an engine or decompressor, or the compressing space of a vacuum pump or compressor) to the low-pressure area (the outlet of an engine or decompressor, or inlet of a compressor or vacuum pump). Therefore, it does not participate in the thermodynamic process that is performed in the chamber. Part of it enters the chamber from those areas, in which gas pressure is higher than that in the chamber at the moment; yet another part leaves it. The working medium contained within constitutes a variable-mass thermodynamic system. The type of process performed by the system depends on the amount of medium flowing in and out through the gaps and on the system's thermodynamic parameters.

Figure 1 depicts the main paths of gas flow in a multi-vane machine. The aim of the present work is to propose a method of determining the mass fluxes for the individual leaks and, consequently, the mass of the gas contained in the chamber in its arbitrary position. The convention employed to describe a position of the chamber and gaps, as well as flow processes themselves, corresponds to that adopted in [1].

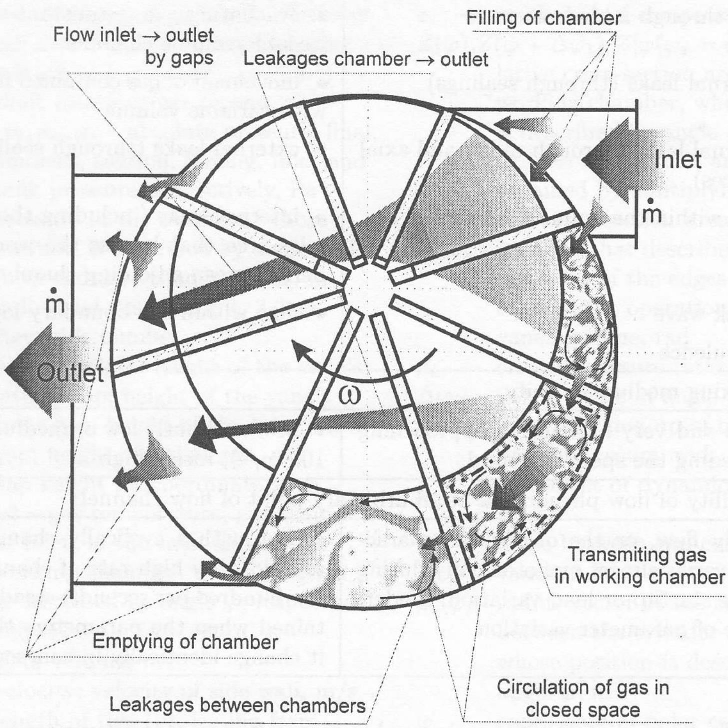

Figure 1. Flow phenomena in multi-vane expansion machine.

2 The method of determining the amount of medium passing through gaps

Parts of a vane rotational machine, while moving with respect to each other, form a number of gaps which differ in shape and character of movement exhibited by their constituting elements. Different can also be the states of medium at opposite ends of the channels. Figure 2 show the most common types of gaps.

The gap in Fig. 2a is formed by the stationary side cover and rotor front. It has the shape of a flat disk or ring with one end rotating. Its distinctive feature is the presence of a number (usually equal to the number of vanes) pressure zones along its circumference. Figure 2b shows the gap between the flat side cover and the side of a vane that rotates in the rotor. The gap forms a channel with a rectangular cross-section, the channel side traveling at a speed depending on the distance from the rotor axis. The gap in Fig. 2c is often called a nozzle-type gap [2,3], as it is formed by two cylinders with usually different diameters, staying close to or even touching each other. Figs. 2d and 2e depict gaps that arise between the vane front and the cylindrical surface of the rotor (e) or the machine cylinder (d). For the latter, the vane can be pressed against either the cylinder or relieving rings. The gap that arises between the cylinder and those parts of the vane front that do not slide over the ring, can be sizable. The form of such a gap is mainly dictated by the shape of the vane front, which forms during the cylinder-vane wearing-in process. The gaps presented in Figs. 2f and 2g, and to some extent in Fig. 2h, constitute channels generated by vast parallel surfaces situated close to each other. The surfaces can be either cylindrical (f) or planar (g and h). Usually one side of the channel remains stationary.

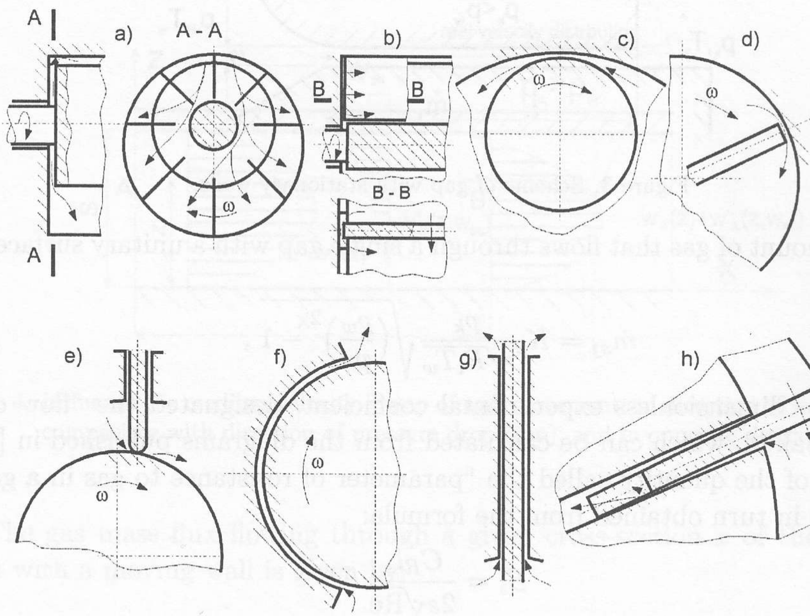

Figure 2. Types of gaps in vane rotational machines.

In the following, length l_s of the gap will be measured in the direction of flow. The width b_s is generally assumed to be its greater (perpendicular to the flow direction) dimension, while height s – the smaller one. Usually $b_s \gg s$. The hydraulic radius of the gap equals then:

$$d_{Hs} = \frac{4b_s s}{2(b_s + s)} \approx 2s$$

and the gap relative length:

$$l_{sw} = l_s / d_{Hs} \, .$$

It should be noted that some of the channels listed above can be continuously or cyclically filled with solid lubricants [4,5], which modify their geometrical characteristics.

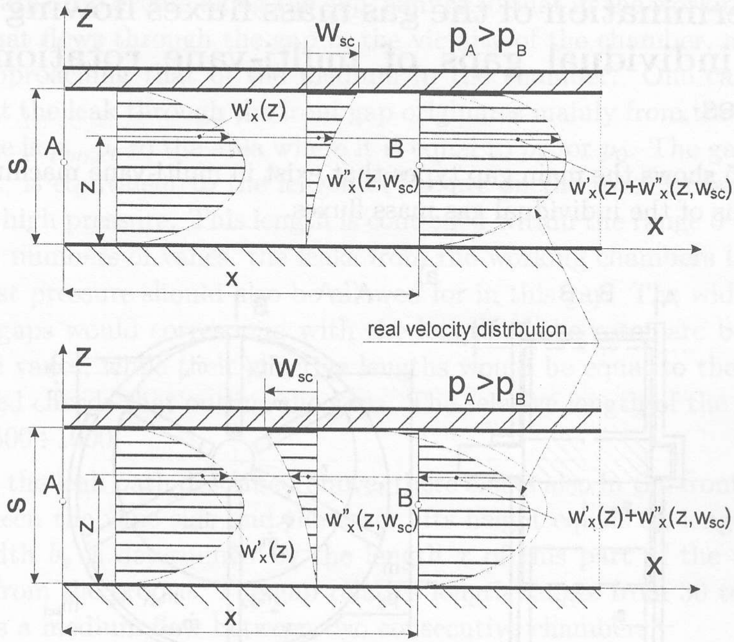

Figure 4. Influence the medium velocity in gap of wall movement: in direction of wall movement compatible with direction of pressure drop (top), and in opposite direction (down).

The gas mass flux flowing through a given cross-section x of the gap (1 m wide) with a moving wall is given by:

$$\dot{m}_{sjbr} = \rho \int_0^s w_x(z)dz = \frac{p}{R_iT}\int_0^s w'_x(z)dz \pm \frac{p}{R_iT}\int_0^s w''_x(z, w_{sc})dz \ . \qquad (2)$$

The "+" sign in this and the following formulae which account for the movement of one of the walls, refers to the case when the sense of the wall velocity vector is identical to that of the medium.

Having allowed for (1), and after appropriate transformations, formula (2) assumes the form:

$$\dot{m}_{sjbr} = \dot{m}_{sj} \pm \frac{1}{2}\frac{ps}{R_iT}w_{sc} \ . \qquad (2a)$$

It can be easily seen from the equation that the gas mass flux flowing through the gap can either be greater or smaller with respect to the case of stationary-wall channel.

3 Determination of the gas mass fluxes flowing through the individual gaps of multi-vane rotational machines

Figure 5 shows the main gap types that exist in multi-vane machines, as well as flow paths of the individual gas mass fluxes.

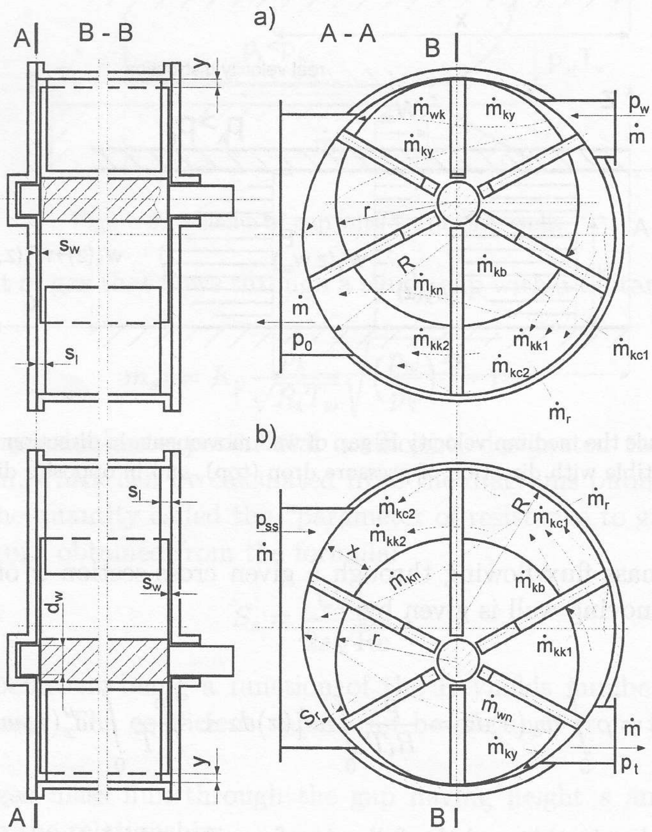

Figure 5. Gap types and leakage paths of gas in multi-vane machines; a) expansion machines, b) compression machines.

The major leak path is through the gap between the rotor front and the side cover. The gap height is s_w, the length l_{sw} depends on the areas it connects. Through this main gap, gas flows mainly from:

- the high-pressure area (p_w, p_t) to the low-pressure area (p_{ss}, p_0), additionally branching off to the working chambers (p_k),
- the working chambers (p_k) to the low-pressure area (p_{ss}, p_0).

A pressure drop occurs along the gap, similar to that in the working chamber. The gas, that flows through the gap in the vicinity of the chamber, assumes the pressure approaching that of the medium in the chamber. One can therefore assume that the leak through the front gap originates mainly from the area where the pressure is p_w, p_t to the area where it is equal to p_{ss} or p_0. The gap width b_s, at the inlet, is equivalent to the length of the arc on the rotor surface, which is exposed to high pressure. This length is contained within the range $0 < l_{sw} < 2r$. For greater numbers of vanes, the leaks from the working chambers to the areas of the lowest pressure should also be allowed for in this gap. The widths of their respective gaps would correspond with the length of the rotor arc between two consecutive vanes, while their effective lengths would be equal to the average of the imagined chords that outline the gaps. The relative length of the gaps lies in the range 500÷1000.

Besides the leak path described above, there exists also in the front cover area a gap between the vane side and the cover. Its height equals s_l, length – $l_s = b$, and its width b_s is determined by the length x of this part of the vane which protrudes from the groove. The gap relative length ranges from 30 to 100. This gap enables a medium flow between two consecutive chambers.

In many types of machines, especially those with a small number of vanes, the medium passes – through the radial gap y – directly from the high-pressure area to the suction or outlet area. The width of this gap equals the length L of the rotor; its shape is similar to that of the Laval nozzle.

If relieving rings are present, or when the vanes are fixed to the shaft, there can also exist a gap between the vane front and the cylinder bearing surface. This gap width is the chamber length minus the width of the rings; the values for the height and length are s_r and b, respectively.

An additional, wedge-shaped gap, having the average height s_b, exists between the vane and the walls of the groove in which it moves, and through which the medium passes between the working chamber and the after-vane space in the groove, or between two consecutive chambers. This flow is only possible, when the vane does not have a constant height. The length of the gap is variable and depends on the length x of this part of the vane that projects from the groove. The average value of the gap length is expressed as $h - x$; its width is equivalent to the vane length.

Equation (1) will be employed for the calculation of the individual gas mass fluxes. In order to distinguish between the different channels through which the leaks arise, the indexes that describe length l_s or width b_s or the gap, and also coefficient K_p, are provided with the denotation of the leak path. Moreover, if the values result form averaging, the supplementary index sr also appears. The pressure and temperature in the working chamber are assumed to be a function of φ – an angle that describes the chamber position. Below, the relationships will be presented which permit calculation of the gas mass flux associated with the particular gaps.

3.1 The gas mass flux $\dot{m}_{wn}$ flowing from the high-pressure area to the low-pressure area through the front gap

The flow of gas occurs between areas characterized by steady pressure values. "High pressure" means pressure at the inlet of an expansion machine – p_w, or the forcing pressure p_t in a compression machine. Similarly, "low pressure" means pressure p_0 at the outlet of an engine or compressor, or the suction pressure – p_{ss}. Both the average length and the width of the gap depend on the rotor position and vary cyclically with a repetition interval of

$$\tau_z = \frac{1}{z_l n_{ob}}$$

this being equivalent to the variation of angle φ equal to:

$$\Delta\varphi = 2\pi/z_l = \lambda\ .$$

Therefore, by defining two periodic functions dependent on the number of vanes z_l, angle ψ, and angles $\alpha_1 \div \alpha_4$, variations in l_{swnsr} and b_{swnsr} can be associated with the position of an arbitrary working chamber.

By way of geometrical analysis [7], one obtains that

$$l_{swnsr} \approx \frac{r}{2}\Big(2 + \sin\frac{\vartheta_1}{2} + \sin\frac{\vartheta_2}{2}\Big)$$

$$b_{swnsr} \approx r\Big(\cos\frac{\vartheta_1}{2} + \cos\frac{\vartheta_2}{2}\Big) - d_w$$

where:

d_w — diameter of rotor shaft at the point where it crosses the side cover;

ϑ_1, ϑ_2 — central angles which describe those arcs on the rotor circumference which are subjected to pressure other than maximum or minimum.

Figure 6 shows an example of the way the functions vary depending on the position of a selected chamber.

During the chamber's rotation by angle λ, the quantities l_{swnsr} and b_{swnsr} assume a number of constant values.

By applying equation (1), one can write

$$\dot{m}_{wn} = s_w b_{swnsr} K_{pwn} \frac{p_0}{\sqrt{R_i T_w}} \sqrt{\Big(\frac{p_w}{p_0}\Big)^2 - 1} \qquad (3)$$

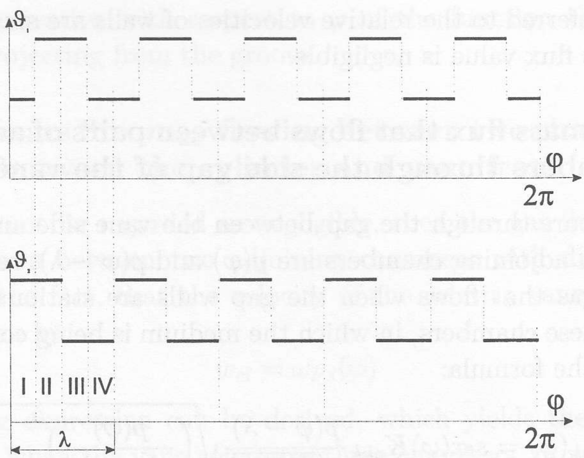

Figure 6. The way the function vary values ϑ_1 and ϑ_2.

or

$$\dot{m}_{wn} = s_w b_{swnsr} K_{pwn} \frac{p_{ss}}{\sqrt{R_i T_t}} \sqrt{\left(\frac{p_t}{p_{ss}}\right)^2 - 1} \tag{4}$$

The value of K_{pwn} for a given gap can be taken from the diagram in [2].

3.2 The gas mass flux m_{kni} flowing from an i^{th} chamber to the low-pressure area

This flux can be determined in a way similar to that presented above by assuming, that the high pressure is the pressure in i^{th} chamber, from which the outflow occurs. The pressure value is $p_i(\varphi)$. Average dimensions of the gap are expressed as:

$$l_{sknisr}(\varphi) = r\left[\sin\frac{j\lambda}{2} + \sin\frac{(j-1)\lambda}{2}\right],$$

$$b_{sknisr}(\varphi) = \frac{1}{4}\lambda r\left[\sin\frac{j\lambda}{2} + \sin\frac{(j-1)\lambda}{2}\right],$$

where j - number of those chambers (preceding the vane, whose position is determined by angle φ), from which the flux m_{kn} flows out.

Allowing for that, one can write

$$\dot{m}_{kni}(\varphi) = s_w b_{sknisr}(\varphi) K_{pkni} \frac{p_0}{\sqrt{R_i T_i(\varphi)}} \sqrt{\left(\frac{p_i(\varphi)}{p_0}\right)^2 - 1} \tag{5}$$

or

$$\dot{m}_{kni}(\varphi) = s_w b_{sknisr}(\varphi) K_{pkni} \frac{p_{ss}}{\sqrt{R_i T_i(\varphi)}} \sqrt{\left(\frac{p_i(\varphi)}{p_{ss}}\right)^2 - 1}\,. \tag{6}$$

In the gaps referred to the relative velocities of walls are small, so their influence on the mass flux value is negligible.

3.3 The gas mass flux that flows between pairs of adjoining working chambers through the side gap of the vane

This flow occurs through the gap between the vane side and the side cover. Pressures in both adjoining chambers are $p(\varphi)$ and $p(\varphi - \lambda)$, respectively.

The flux of gas that flows when the gap walls are stationary and the vane fully separates these chambers, in which the medium is being compressed, can be calculated from the formula:

$$\dot{m}_{kk}(\varphi) = s_l x(\varphi) K_{pkk} \frac{p(\varphi - \lambda)}{\sqrt{R_i T(\varphi)}} \sqrt{\left(\frac{p(\varphi)}{p(\varphi - \lambda)}\right)^2 - 1}\,. \tag{7}$$

When the medium is being expanded in the chambers, then:

$$\dot{m}_{kk}(\varphi) = s_l x(\varphi) K_{pkk} \frac{p(\varphi)}{\sqrt{R_i T(\varphi - \lambda)}} \sqrt{\left(\frac{p(\varphi - \lambda)}{p(\varphi)}\right)^2 - 1}\,. \tag{8}$$

The relative velocity of the vane with respect to the cover is expressed as

$$w_{srl} = \frac{1}{2}\omega[\rho_s(\varphi) + r]\,.$$

By allowing for this formula and (1a) as well, the gas mass flux passing a moving channel can be obtained from:

$$\dot{m}_{kkr}(\varphi) = \left\{1 \pm \frac{1}{4}\frac{\omega[\rho_s(\varphi) + r]}{w_{skk}}\right\} \dot{m}_{kk}(\varphi)\,. \tag{9}$$

The "+" sign in the above expression refers to the situation, in which the direction of flow is consistent with the direction of vane movement (the medium is being expanded).

3.4 The gas mass flux $\dot{m}_{\mathbf{kb}}$ flowing from the chamber to the after-vane space

The flow of gas through the gap in the groove, which connects the higher-pressure chamber with the after-vane space, cannot proceed more quickly than the rate of mass increase in this area due to volume increase. The flux thus estimated won't be exceeded, because the after-vane pressure won't be higher than the pressure in the chamber, from which the gas is flowing. This flux is expressed as:

$$\dot{m}_{kb\,\max} = \omega b L x'(\varphi) \frac{p(\varphi - \lambda)}{R_i T(\varphi - \lambda)}, \tag{10}$$

where $x'(\varphi)$ – derivative with respect to φ of the function that describes the height of vane projecting from the groove.

3.5 The gas mass flux m_{kc} flowing between two chambers through the gap between the cylinder and vane front

If gap s_r cannot be regarded as negligible, then the gas flux has to be considered which flows between two adjoining chambers with different pressures. By allowing for the fact, that the velocity of the relative vane front-to-cylinder movement is given by

$$w_{cl} = \omega\rho_s(\varphi)$$

The following expression can be derived, which yields the gas flux flowing through the gap when the vane separates these chambers, in which compression takes place:

$$\dot{m}_{kcr}(\varphi) = \left[1 - \frac{1}{2}\frac{\omega\rho_s(\varphi)}{w_{skc}}\right] s_r L K_{pkc} \frac{p(\varphi-\lambda)}{\sqrt{R_i T(\varphi)}} \sqrt{\left[\frac{p(\varphi)}{p(\varphi-\lambda)}\right]^2 - 1}\,. \tag{11}$$

Similarly, for the case when the vane separates these chambers, in which gas is being expanded:

$$\dot{m}_{kcr}(\varphi) = \left[1 - \frac{1}{2}\frac{\omega\rho_s(\varphi)}{w_{skc}}\right] s_r L K_{pkc} \frac{p(\varphi)}{\sqrt{R_i T(\varphi-\lambda)}} \sqrt{\left[\frac{p(\varphi-\lambda)}{p(\varphi)}\right]^2 - 1}\,. \tag{12}$$

3.6 The gas mass flux m_{ky} flowing through the radial gap with height y

This sort of flux can be calculated for gaps of the Laval type, using the relationship [3]:

$$\frac{dp}{dx} = \frac{1}{s(x)} \frac{\left[12\dot{m}_{sjb}\eta - \frac{21}{5}\dot{m}_{sjb}^2\left(\frac{x}{\sqrt{r^2-x^2}} - \frac{x}{\sqrt{R^2-x^2}}\right)\right]p}{\left\{\frac{6}{5}\frac{\dot{m}_{sjb}^2}{n} - \frac{1}{z_s R_i T}[s(x)]^2 p^2\right\}} \tag{13}$$

in which geometrical parameters $s(x)$ and x, as shown in Fig. 7, appear.

The analytical form of the function $s(x)$ is as follows:

$$s(x) = \sqrt{R^2 - x^2} - \sqrt{r^2 - x^2} - e\,.$$

The gap length l_{sky} equals $2\pi r/z_l$. The flux value $\dot{m}''_{sjb}$, obtained from Eq. (13), will allow to calculate $\dot{m}''_{ky}$:

$$\dot{m}_{ky} = \dot{m}_{sjb} L\,. \tag{14}$$

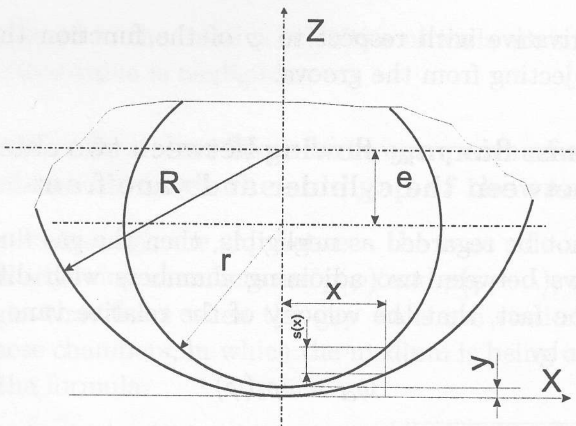

Figure 7. Relationship of gap height for its length for eccentric circles.

By allowing for the movement of the rotor surface with respect to the cylinder one obtains:

$$\dot{m}_{kyr} = \left[1 \pm \frac{1}{2}\frac{\omega r}{w_{ky}}\right]\dot{m}_{ky} \,. \tag{14a}$$

4 Mass of the gas contained in the working chamber of a multi-vane rotational machine during one full cycle of operation

For simplicity sake, the analysis of the working chamber filling level in multi-vane expansion-type machines is usually started by assuming such a working chamber position, which correspond to the angle $\varphi \approx -\lambda/2$.

If $\alpha_1 < \lambda/2$, then the gas which fills the chamber has its inlet parameters. This case is discussed further. If, however, $\alpha_1 \geq \lambda/2$ is assumed, than the pressure and temperature of the gas in the chamber result from earlier processes, so for the chamber position characterized by the angle $\varphi = \varphi(\varphi_s = \alpha_1) - \lambda$, they are expressed as $p[\varphi(\varphi_s = \alpha_1) - \lambda]$ and $T[\varphi(\varphi_s = \alpha_1) - \lambda]$, respectively; the mass of the gas is given as:

$$m[\varphi(\varphi_s = \alpha_1) - \lambda] = R^2 LZ[\varphi(\varphi_s = \alpha_1) - \lambda]\frac{p[\varphi(\varphi_s = \alpha_1) - \lambda]}{R_i T[\varphi(\varphi = \alpha_1) - \lambda]} \,. \tag{15}$$

In the process of further rotation of the rotor, the "initial" vane of the chamber exposes the inlet edge and initiates the phase of intentional unsealing of the working space, during which the filling of the chamber occurs. The first moment of this process is accompanied by the necessary equalization of pressures in the working chamber and the inlet area. Most often $p_z(\varphi) < p_w$, which means that "loading" of the chamber takes place.

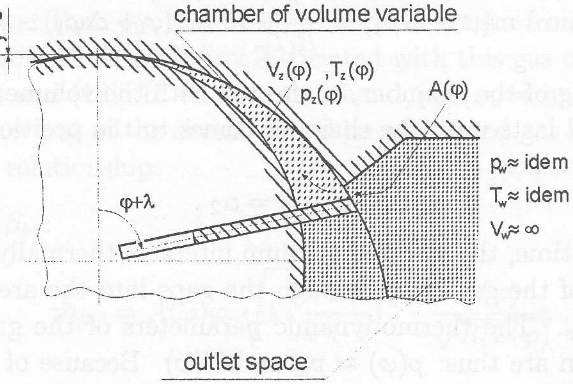

Figure 8. "Loading" the working chamber.

By treating the channel, which connects the inlet area with the chamber, as a convergent nozzle (see Fig. 8), the process can be regarded as an adiabatic (although not isentropic) gas flow through a nozzle into a variable-volume tank. The flux of gas that enters the tank depends on the ratio p_z/p_w and on the minimal cross-section area of the nozzle $A(\varphi)$. Pressure p_w is usually steady and $p_z(\varphi)$, $T_z(\varphi)$, $V_z(\varphi)$, and $A_z(\varphi)$ are some functions of position φ.

If $p_z/p_w \leq \beta_{kr}$, then:

$$\dot{m}_{kd} = A(\varphi)\varphi_p\sqrt{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}\frac{p_w}{\sqrt{R_iT_w}}\,, \tag{16}$$

where φ_p – velocity coefficient, depending on the nozzle shape.

If $p_z/p_w > \beta_{kr}$, then the flux $\dot{m}_{kd}$ is given by:

$$\dot{m}_{kd} = A(\varphi)\varphi_p\sqrt{\frac{2k}{k-1}\left[\left(\frac{p_z(\varphi)}{p_w}\right)^{\frac{2}{k}} - \left(\frac{p_z(\varphi)}{p_w}\right)^{\frac{k+1}{k}}\right]}\frac{p_w}{\sqrt{R_iT_w}}\,. \tag{17}$$

The overall amount of medium in the chamber for an arbitrary moment from the beginning of "loading" is:

$$m(\varphi) = m[\varphi(\varphi_s = \alpha_1) - \lambda] + \frac{1}{\omega}\int_{\alpha_1}^{\varphi}\dot{m}_{kd}(\varphi)d\varphi\,. \tag{18}$$

If $A(\varphi)$ is sufficiently high, then the angle $\Delta\varphi_1$, during which the "loading" takes place, is small. This often leads to the assumption that the pressure equalization occurs immediately.

When the inlet pressure p_w is attained in the chamber, the amount of gas present in it is equal to:

$$m(\varphi + \Delta\varphi_1) = \frac{p_w}{R_i T_w} R^2 L Z(\varphi + \Delta\varphi_1)\,. \tag{19}$$

Further filling of the chamber, connected with the volume being increased, is much slower and lasts until the chamber comes to the position characterized by the angle φ, for which:

$$\varphi_s(\varphi) = \alpha_2\,.$$

At the same time, the working medium interacts thermally with the chamber walls and part of the gas flows through the gaps into the areas, where pressure is lower than p_w. The thermodynamic parameters of the gas for an arbitrary chamber position are thus: $p(\varphi) \approx p_w$ and $T(\varphi)$. Because of the heat exchange and gas escape through the gaps, this temperature differs from T_w. The amount of medium contained in the chamber:

$$m(\varphi) = R^2 L Z(\varphi) \frac{p_w}{R_i T(\varphi)}\,. \tag{20}$$

The closing of the chamber initiates the expansion phase. It is started by the gas closed in the chamber at the end of the filling process. The gas mass is given by:

$$m[\varphi_s(\varphi) = \alpha_2] = R^2 L Z(\varphi_s = \alpha_2) \frac{p_w}{R_i T(\varphi_s = \alpha_2)}\,. \tag{21}$$

During expansion, i.e. when the chamber assumes positions described by the angle φ which fulfills the inequality

$$\alpha_2 < \varphi_s(\varphi) < \alpha_3 - \lambda$$

the mass of the gas in the chamber changes due to the flow through gaps. It is expressed for an arbitrary moment by:

$$m(\varphi) = m[\varphi(\varphi_s = \alpha_2)] + \frac{1}{\omega} \int\limits_{\varphi(\varphi_s=\alpha_2)}^{\varphi} \sum_i \dot{m}_{id}\, d\varphi - \frac{1}{\omega} \int\limits_{\varphi(\varphi_s=\alpha_2)}^{\varphi} \sum_j \dot{m}_{jw}\, d\varphi\,, \tag{22}$$

where:

$\sum_i \dot{m}_{id}$ – the sum of all the i mass fluxes entering the working chamber $[\dot{m}_{kkr}(\varphi),\ \dot{m}_{kcr}(\varphi), \ldots]$;

$\sum_j \dot{m}_{jw}$ – the sum of all the j mass fluxes leaving the chamber $[\dot{m}_{kn}(\varphi),\ \dot{m}_{kkr}(\varphi + \lambda),\ \dot{m}_{kcr}(\varphi + \lambda),\ \dot{m}_{kb\,\max}]$;

$\varphi(\varphi_s = \alpha_2)$ – the angle describing such a position of the chamber, that $\varphi_s = \alpha_2$.

When the vane exposes the edge associated with the angle α_4, the working chamber and outlet area become connected. An appropriately selected angle α_4 ensures, that $p[\varphi(\varphi_s = \alpha_3) - \lambda] = p_0$. If, due to a design error, pressure $p[\varphi(\varphi_s = \alpha_3) - \lambda] = p_0$ is lower than p_0, the medium tends to return from the outlet area back into the chamber. The mass flux associated with this gas can be calculated from formulae (17) and (18). If, on the other hand, $p[\varphi(\varphi_s = \alpha_3) - \lambda] > p_0$, then the medium will flow out of the chamber and its associated gas mass flow can be obtained from the relationship:

- if $p_0/p(\varphi) < \beta_{kr}$:

$$\dot{m}_{kw} = A(\varphi)\varphi_p\sqrt{k\Big(\frac{2}{k+1}\Big)^{\frac{k+1}{k-1}}}\frac{p(\varphi)}{\sqrt{R_iT(\varphi)}}, \tag{23}$$

- or, if $p_0/p(\varphi) > \beta_{kr}$:

$$\dot{m}_{kw} = A(\varphi)\varphi_p\frac{p(\varphi)}{\sqrt{R_iT(\varphi)}}\sqrt{\frac{2k}{k-1}\Big[\Big(\frac{p_0}{p(\varphi)}\Big)^{\frac{2}{k}} - \Big(\frac{p_0}{p(\varphi)}\Big)^{\frac{k+1}{k}}\Big]}. \tag{24}$$

The quick increase of $A(\varphi)$ leads to a rapid decrease of the chamber-opening criterion and, consequently, the discharge or loading occurs very quickly (since $\Delta\varphi_2$ is small). It is therefore often assumed that this process is instantaneous. At the end of it, the amount of medium in the chamber is:

$$m(\varphi + \Delta\varphi_2) = \frac{p_0}{R_iT(\varphi + \Delta\varphi_2)}R^2LZ(\varphi + \Delta\varphi_2). \tag{25}$$

When the working chamber turns from $\varphi = \varphi(\varphi_s = \alpha_3) - \lambda$ to $\varphi = \varphi(\varphi_s = \alpha_4)$, i.e. during the ejection phase, the pressure within it remains constant at the level of p_0. The temperature, however, depends on the thermal interaction with the chamber walls. The amount of gas contained in the chamber for any chamber position in the interval of φ values discussed, can be calculated from the formula:

$$m(\varphi) = R^2LZ(\varphi)\frac{p_0}{R_iT(\varphi)}. \tag{26}$$

During the final stage of rotor rotation, and when $y > 0$, the compression phase is effected, which lasts when the chamber passes through positions described by angle φ in the range $\varphi(\varphi_s = \alpha_4) < \varphi < \varphi(\varphi_s = \alpha_1) - \lambda$. The amount of working medium in the chamber is then determined by the relation:

$$m(\varphi) = m[\varphi(\varphi_s = \alpha_4)] + \frac{1}{\omega}\int\limits_{\varphi(\varphi_s=\alpha_4)}^{\varphi}\sum_l \dot{m}_{ld}d\varphi - \frac{1}{\omega}\int\limits_{\varphi(\varphi_s=\alpha_4)}^{\varphi}\sum_k \dot{m}_{kw}d\varphi\ , \tag{27}$$

where:

$\sum\limits_{l} \dot{m}_{ld}$ – the sum of all the l gas fluxes entering the chamber at this phase,

$\sum\limits_{k} \dot{m}_{kw}$ – the sum of all the k fluxes leaving the chamber.

After connecting the chamber with the inlet area $(\varphi > \varphi(\varphi_s = \alpha_1) - \lambda$ the cycle repeats itself.

The most suitable starting moment for the investigation of the amount of gas in the working chamber of a compression machine is when, after having been filled, the chamber has just been closed, i.e. for $\varphi = \varphi(\varphi_s = \alpha_4)$. The mass of the medium is then expressed as

$$m[\varphi(\varphi_s = \alpha_4)] = R^2 LZ[\varphi(\varphi_s = \alpha_4)] \frac{p_{ss}}{R_i T[\varphi(\varphi_s = \alpha_4)]} \,. \tag{28}$$

For $\varphi(\varphi_s = \alpha_4) \leq \varphi < \varphi(\varphi_s = \alpha_1) - \lambda$, this can be transformed into:

$$m(\varphi) = m[\varphi(\varphi_s = \alpha_4)] + \frac{1}{\omega} \int\limits_{\varphi(\varphi = \alpha_4)}^{\varphi} \sum_i \dot{m}_{id} d\varphi - \frac{1}{\omega} \int\limits_{\varphi(\varphi = \alpha_4)}^{\varphi} \sum_j \dot{m}_{jw} d\varphi \,. \tag{29}$$

During the further rotation of the chamber by the angle of $\Delta\varphi_3$, the chamber is opened and the pressure equalized. Both fluxes – the one which flows into and the one which flow out of the chamber – can be calculated from formulae (16), (17), (23), (24). In some cases, i.e. when $\Delta\varphi_3 \to 0$, the processes are regarded as instantaneous.

When $[\varphi(\varphi_s = \alpha_1) - \lambda] + \Delta\varphi < \varphi < \varphi(\varphi_s = \alpha_2)$, then:

$$m(\varphi) = R^2 LZ(\varphi) \frac{p_t}{R_i T(\varphi)} \,. \tag{30}$$

When $y > 0$ and $\varphi(\varphi_s = \alpha_2) < \varphi < \varphi(\varphi_s = \alpha_2) - \lambda$, then during expansion:

$$m(\varphi) = m[\varphi(\varphi_s = \alpha_2)] + \frac{1}{\omega} \int\limits_{\varphi(\varphi_s = \alpha_2)}^{\varphi} \sum_l \dot{m}_{ld} d\varphi - \frac{1}{\omega} \int\limits_{\varphi(\varphi_s = \alpha_2)}^{\varphi} \sum_k \dot{m}_{kw} d\varphi \,. \tag{31}$$

When the chamber is opened during its rotation by $\Delta\varphi_4$, the equalization of pressure takes place in the chamber and suction area. If $\Delta\varphi_4 \to 0$, the process can be treated as immediate. From the moment the pressures are equalized and the chamber fully unsealed, the phase of filling the working space starts, which lasts until $\varphi = \varphi(\varphi_s = \alpha_4)$. The amount of medium in the working chamber during that interval can be determined from the equation:

$$m(\varphi) = R^2 LZ(\varphi) \frac{p_{ss}}{R_i T(\varphi)} \,. \tag{32}$$

When the chamber is closed again $[\varphi = \varphi(\varphi_s = \alpha_4)]$, a new cycle of operation begins.

By employing Eqs. (3-14) and Eqs. (18-22) and (25-32), the mass of gas for the working chamber's arbitrary position can be calculated. The essential parameters that permit to find out the mass of medium are the medium's thermodynamic parameters – pressure $p(\varphi)$ and temperature $T(\varphi)$. In [8], the author proposed relationships that allow to determine them.

A first approximation in these calculations can be the assumption that these parameters change as they would in an ideal machine (e.g. adiabatic compression and decompression of a constant gas amount in hermetic chambers, and isobaric-isothermic intake and ejecting the medium).

Besides determining the absolute mass of gas, the necessity sometimes arises to determine the relative gas amount. Assuming as a reference – for example – the mass of gas contained in the working chamber at the moment it is being closed (i.e. $[m[\varphi(\varphi_s = \alpha_2)]] = m_w$), the relative amount of gas in the chamber $\overline{m}$, as defined by the ratio

$$\overline{m} = \frac{m(\varphi)}{m_w} \tag{33}$$

can be investigated.

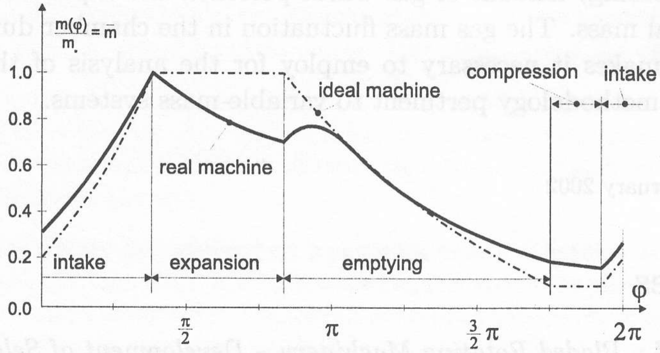

Figure 9. Relationship of relative mass of gas in the chamber for position angle.

Figure 9 shows a sample relation between $\overline{m}$ and angle φ for an expansion machine. The calculations were carried out for specification: $s_l = s_w = 0.03$ mm, $s_R = 0$; $R = 60$ mm, $r = 58$ mm, $L = 120$ mm.

It follows from the diagram that when the gas contained in the working chamber performs the expansion process, its mass decreases. In calculations carried out by the Author, the relative gas mass at the end of the process covered the interval of $\overline{m} = 0.9 \div 0.7$, sometimes even less, depending on gap sizes, medium inlet parameters and geometrical dimensions of the machine. For medium masses fluctuating so strongly, relationships derived for the case of the constant gas mass can lead to considerable errors, if applied to the description of thermodynamic processes of that type.

5 Final remarks

In the paper, a characterization of medium flow through volumetric rotational machines is attempted. Taking a multi-vane machine as an example, leaks paths and transport modes of the medium from the inlet area to the outlet area are shown. It is pointed out that the working chamber is by no means the only place for such a flow to occur.

Discussed is a method of calculating the amount of working medium, which flows through gaps with both stationary and moving walls. The characteristic of gaps which are present in multi-vane machines is given and analytical formulae proposed that allow to determine the individual gas mass fluxes.

Based on the formulae given, the amount of medium in the working chamber is calculated during its full rotation and the $\overline{m}$ quantity defined, which indicates the relative mass of medium in the chamber. By visualizing, for a selected set of data, this dependence on chamber position, it was possible to show the strongly varying (decreasing) amount of gas which performs the expansion process - up to 0,7 of initial mass. The gas mass fluctuation in the chamber during the phase of expansion makes it necessary to employ for the analysis of thermodynamic processes the methodology pertinent to variable-mass systems.

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