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Flow behaviour of ice slurry – experimental investigation

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Abstract

The paper presents results of studies on ice slurry flows through horizontal tubes. The possibility of treating rheological parameters of ice slurry as being those of a kind of Bingham fluid was confirmed. The values of parameters (mass fraction, flow velocity) corresponding to laminar, intermediate and turbulent flow, were determined. This permits to optimize the flow in the systems working with ice slurries. The work includes a comparison of experimental results with computational methods of determining pressure drops for a Bingham fluid over laminar (the Hedström method) and turbulent (the Tomita method) ranges. A verification, based on author's experimental results, of the Fanning and Blasius expression for the Fanning friction factor of ice slurry became possible by introducing the dimensionless generalised Reynolds number. In the last part of the paper the issues connected with the criteria for transition of ice slurry from laminar to turbulent flow are discussed.

Keywords: Ice slurry; Momentum transfer; Experimental investigation

Nomenclature

a – coefficient in the polynomial function (Eq. 4)
c_f – Fanning friction factor
d – diameter, m
He – Hedström number, $\frac{d^2 \tau_p \rho}{\mu_p^2}$
K – consistency index, $\mathrm{Ns}^n/\mathrm{m}^2$
L – tube length, m
$\dot{m}$ – mass flow rate, kg/s
n – characteristic flow-behaviour index, $\frac{d(\ln \tau_w)}{d(\ln \Gamma)}$
p – pressure, Pa
Re – Reynolds number, $\frac{dw\rho}{\mu}$

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w, wm	–	flow velocity, m/s
x_a, xa	–	carrying fluid concentration, %
x_s, xs	–	mass fraction of ice, %
ε_B	–	quotient $\frac{\tau_p}{\tau_w}$
$\dot{\gamma}$	–	shear rate, s^{-1}
Γ	–	tube shear rate, s^{-1}
μ	–	dynamic viscosity, Pa s
μ_p	–	plastic viscosity, Pa·s
ρ	–	density, kg/m ^{3}
τ	–	shear stress, Pa
τ_p	–	yield (plastic) shear stress, Pa
τ_w	–	shear stress on tube wall, Pa
τ_{wL}	–	shear stress on tube wall for laminar flow, Pa

Subscripts

a	–	carrying fluid
B	–	Bingham fluid
C	–	critical value
i	–	internal; initial
i, j	–	indices
m, $mean$	–	mixture; mean value
MR	–	Metzner–Read
MM	–	Matras
M	–	Gücüyener and Mehmetoglu
s	–	ice; solid particle
T	–	turbulent

1 Introduction

Ice slurries belong to a group of refrigerating media. They are used in cooling and air-conditioning technology, mostly in heat storage systems. Ice slurry is a mixture of ice-water (with a maximum solid particle size of d_s<0.5 mm) and water, with an addition of a substance lowering the freezing point (such as salt, glycol, alcohol). Ice slurries have been the object of research for over 10 years now. However, it is clear from the analysis of available literature that the scope of research is limited, both in terms of geometrical parameters of the tested systems and the variability of thermal-flow parameters. Such limitations make the application range of the results obtained from experimental research rather narrow, because they do not allow to fully identify the investigated phenomena [1, 3, 8, 9, 14, 18, 21, 26, 30]. Ice slurry is not a Newtonian liquid. Its non-Newtonian behaviour is more pronounced for larger contents of solid particles (x_s > 10%). In most cases plastic–viscous fluid models are ascribed to ice slurry, namely the Oswald de Waele fluid model [12, 19], the Bingham model [3, 8, 9, 14], and the Casson model [6]. Time dependent behaviour of ice slurry was the subject of investigations presented in [16]. The Papanastasiou model [25] is a model of rheologically unstable fluid (time-dependent non-Newtonian fluid), ascribed to ice slurries. There are few papers which treat ice slurry as a multiphase medium [1, 16, 17, 24]. The Bingham model is most often used to characterize this fluid. To identify Bingham plastics from the rheological point of view it is necessary to determine their plastic viscosity μ_p and yield shear stress τ_p. Generally, there is a lack of such data in the literature. The existing τ_p data [3, 9, 14, 26] are often contradictory. Pressure drops, calculated based on these data (the Hedström method), are significantly overestimated. The values of τ_p and μ_p depend on the size of solid particles and the geometry of the resulting structures. Therefore, τ_p and μ_p results may vary, depending, for example, on the method of ice production.

The present paper, based on the author's experimental results, focuses on systematization and explanation of various phenomena which accompany ice slurry flows in straight tubes, as the data published in the literature are often divergent [1]. The work determines rheological properties of ice slurry with the particles of the size of $0.05 \leq d_s \leq 0.35$ mm, and produced from ethanol solution with the initial concentration of $x_{ai} = 10.6\%$. Selecting a single initial solution concentration, typical of technical applications, allowed to perform a large number of measurements for different ice contents and different pipe diameters. At the same time, it permitted to determine critical values of Reynolds numbers, as well *as to observe the effect of intersection of the carrying liquid resistance curve with the ice slurry resistance curve.*

The work includes a comparison of experimental results with the results of classical methods of pressure drops determination for ice slurries, treated as Bingham fluids (the Hedström method and the Tomita method). A verification, based on the author's experimental results, of the Fanning and Blasius expression for the Fanning friction factor for ice slurry was made possible by introducing the dimensionless generalised Reynolds number. In the final part of this paper the criteria of the change of ice slurry flow character are discussed.

2 Ice slurry flow in straight tubes – experimental investigation

The measurements were performed using the test stand shown in Figure 1. The Energietechnik Integral T 100T ice generator with a modified mixing device in the accumulation container was used in the system. The second, fundamental part of the test stand enables very accurate measurements of thermal and flow parameters using the equipment listed in Tab. 1.

Measured values are recorded by a data acquisition system. An exchangeable segment of the installation, of the length of 4.6 m, allowed measuring thermal and flowing parameters for tubes with various diameters. This segment was long

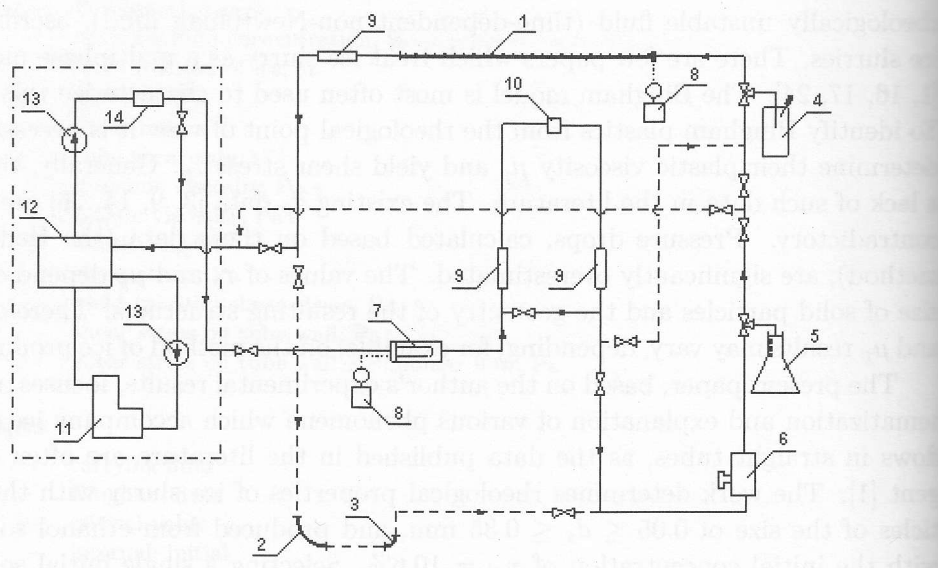


Figure 1. Schematic diagram of the test stand: 1 – heated test tube, 2 – test elbow, 3 – test bend, 4 – calorimetric measurement, 5 – measurement of ice density and air concentration, 6 – mass flow-meter, 7 – heater, 8 – autotransformer and wattmeter, 9 – visualization, 10 – air-escape, 11 – ice slurry generator, 12 – ice storage, 13 – pump, 14 – volume flow-meter.

enough to enable for complete formation of the flow.

In the present studies, the mass fraction of ice was determined based on two parameters, measured simultaneously and in a continuous manner, i.e. temperature (temperature sensor Pt 100 7013) and mixture density (mass flowmeter Danfoss 2100). Good reproducibility of these parameters made it possible to determine the fraction of ice over the whole measuring range, with the difference between the results of various methods not exceeding 1.5%. Ice content was checked at selected points using a calorimetric method, while the amount of air present in the ice slurry (especially at higher x_s values) was checked by a volumetric method. A divergence of the temperature- and density-derived results indicates a non-homogeneity or aeration of the mixture, and – for a longer time – a change of concentration. To measure the aeration and for calorimetric measurements a quick-switching valve was used. Plexiglass tubes, placed at the outlet of the measuring section and on vertical sections, were used for visualization of the flow. Differential pressure measurements, necessary for determination of pressure drops, were carried out by means of differential pressure transducers FKCV made by Fuji, with a measuring range adjusted to actual pressure drops. Due to ice slurry properties, the pressure tapping in the installation was performed by an impulse connector with a relatively large diameter; circumferential groove was

Table 1. List of the most important measuring devices.

Device	Type	Range	Accuracy
Differential Pressure transducers	Fuji FKCV 33V4LKAYYAA Fuji FKCV 11V4LKAYYAA	$0 \div 32$ kPa $0 \div 1$ kPa	±0.07% of measuring range
Temperature sensors	HART Scientific Pt100(7013) ϕ 5 mm Pt100(5622-05) ϕ 0.5 mm	$-100 \div 100$°C $-200 \div 350$°C	±0.018°C ±0.15°C nominally
Mass (density) flowmeter	Danfoss 2100	$0 \div 5600$ kgh^{-1} $0.1 \div 2.9$ g cm^{-3}	±0.1% of flow ±0.0005 g cm^{-3}

not used. The connectors were 4.0 mm of inner diameter and were made in a way that facilitated automatic venting of the impulse conduits, which were not insulated. The research project included the measurements for:

- Copper tubes with diameters of $d_i = 0.01$; 0.016 and 0.02 m.
- Mean flow velocities of $0.1 \le w_m \le 4.5$ m/s, with the corresponding Reynolds number values in the range: $300 < \mathrm{Re}_B < 12\,000$.
- Mass fractions of ice of $0 \le x_s < 30\%$, with an average ice crystal size of $d_s = 0.165/0.235$ mm (width/length) [7]. Measurements were performed, for "old" ice, under controlled conditions to ensure the repeatability and coherence of the obtained results.

Systematic errors of measured values are presented in Tab. 2.

Table 2. Systematic errors of measured values.

	x_s	$w_m \cdot 10^3$ m s^{-1}	$\mu_p \cdot 10^4$ Pa s	$\tau_p \cdot 10^2$ N m^{-2}
Absolute error	0.06...0.11	1.8...100	1.3...1.8	4.8...7.5
	1.7...4 %	0.9...2.1%	0.8...2.4%	3.5...7.4%
	ρ kg m^{-3}	Re	He	$c_f \cdot 10^3$
Absolute error	0.5	4.5...890	0...364	0.25...34
Percentage error	0.5%	3...8.4%	10.3...14.9%	2.4...5.8%

Figure 2 shows the values of pressure drops as a function of mean velocity. As can be seen, for each ice content higher than the limiting value, there exist velocity

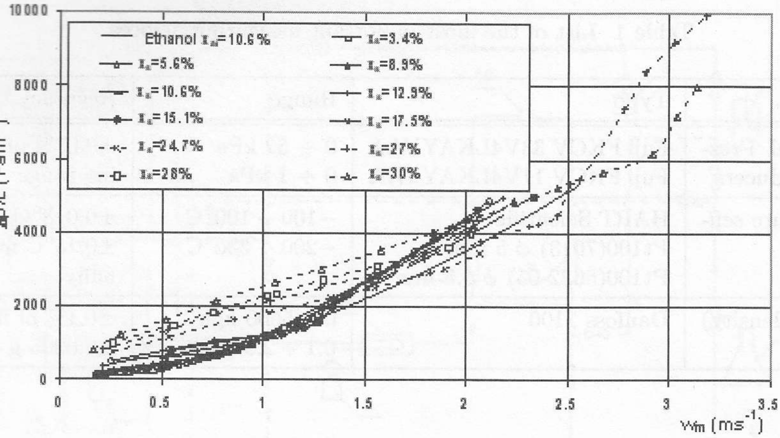


Figure 2. Pressure drop $(\Delta p/L)$ as a function of mean flow velocity w_m of ice slurry for $d_i =$ 0.016 m.

regions in which pressure drops are lower than the pressure drops of the carrying fluid flow (ethanol solution with $x_{ai} = 10.6\%$ at $-4.5°C$). This effect is observed for all tube diameters studied in this work. It is least visible for $d_i = 0.01$ m. Based on the function $\Delta p\,(w_m)_{d=const}$, the so-called flow curves $\tau_w(\Gamma)$ (Fig. 3) were prepared, where [20]:

$$\tau_w = \frac{d_i \cdot \Delta p}{4 \cdot L}, \tag{1}$$

$$\Gamma = \frac{8 \cdot w_m}{d_i}. \tag{2}$$

For the studied refrigerant, independent of the tube diameter, the curves obey the Bingham rheological model [20]:

$$\tau = \tau_p + \dot{\gamma} \cdot \mu_p \quad \Longrightarrow \quad \tau_w = \frac{4}{3}\tau_p + \mu_p \Gamma. \tag{3}$$

Flow curves (3) made it possible to determine the experimental values of plastic viscosity (μ_p) and yield shear stress (τ_p) for ice slurries with various mass fractions of ice. The obtained results $(\tau_p,\, \mu_p)$ were approximated by the following polynomial function:

$$y = \sum_{i=0}^{5} a_i \left(\frac{x_s}{100}\right)^i \quad \text{for} \quad 0\% < x_s < 30\%. \tag{4}$$

The values of the a_i coefficients for τ_p and μ_p are presented in Tab. 3.

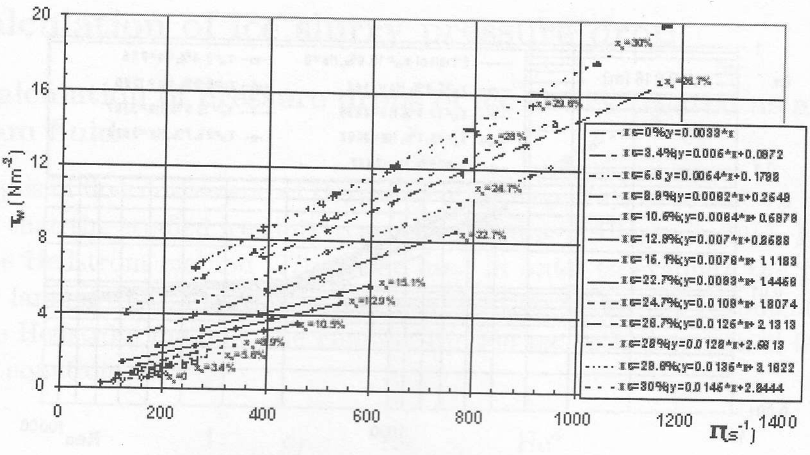


Figure 3. Flow curves for $d_i = 0.016$ m.

Table 3. Coefficients of the polynomial function (4).

	a_0	a_1	a_2	a_3	a_4	a_5
μ_p	0.0035	0.0644	–0.7394	5.6963	–19.759	26.732
τ_p	0.013	–1.4284	73.453	–394.64	835.82	0

The above values (τ_p, μ_p,) together with ice slurry density ρ_B [2], allowed an evaluation of the Reynolds and Hedström numbers for a Bingham fluid as well as of the Fanning friction factor c_f:

$$c_f = \frac{d_i \cdot \Delta p}{2 \cdot L \cdot \rho_B \cdot w_m^2}. \tag{5}$$

The dependence $c_f = (\mathrm{Re}_B, \mathrm{He})$ for $d_i = 0.016$ m is shown in Figure 4. The non-monotonic character of $c_f = (\mathrm{Re}_B)$ function in $c_f = (\mathrm{Re}_B, \mathrm{He})$ plots clearly shows that the laminar flow loses its stability. In the experiments this was reflected by unstable pressure readings. The laminar – intermediate flow transition was observed within the range of $1800 < \mathrm{Re}_B < 3000$.

The plots shown in Fig. 5 illustrate the pressure drops as a function of ice content at steady flow velocities. The symbol "–" on the Y-axis in Figure 5 denotes the values of pressure drops of ethanol solution (the carrying fluid) at selected velocities. The plot displays the experimental line depicting the change of the flow character. An analysis of the $\Delta p/L(x_s)_{wm}$ curves shows that:

- for ice content $x_s \geq 10\%$ and the same flow velocity, pressure drops of

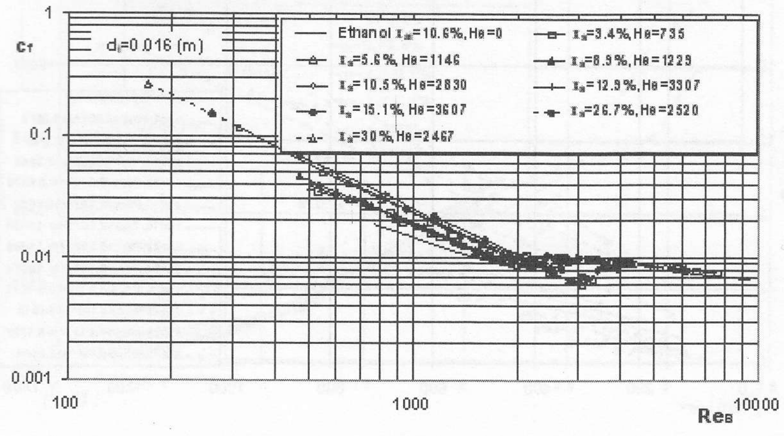


Figure 4. Experimental values of friction factor c_f as function of Re_B and He for $d_i = 0.016$ m.

ice slurry can be lower than those of the carrying fluid, provided that the flow character is different for ice slurry and the carrying fluid;

- except for the intermediate flow region, the function $\frac{\Delta p}{L}(x_s)$ is an increasing function;
- the change of $\Delta p/L$ with x_s is more pronounced in the laminar region;
- at flow velocities $w_m < 0.5$ m s^{-1}, the flow is laminar for all values of x_s.

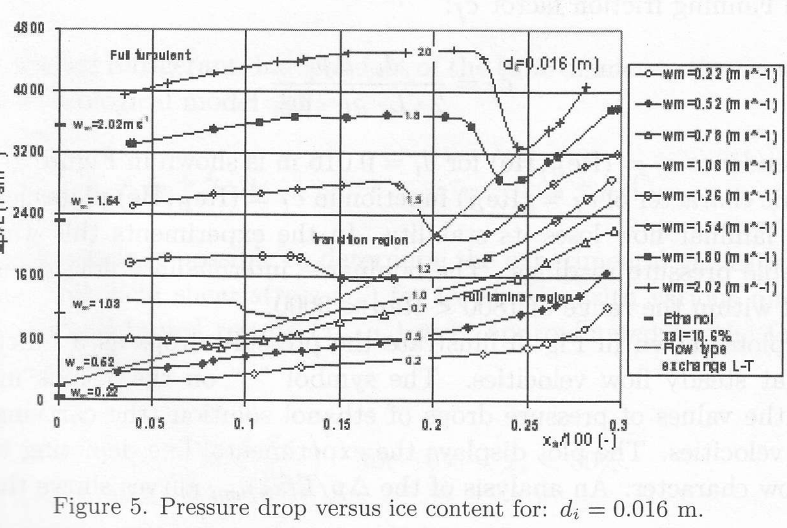


Figure 5. Pressure drop versus ice content for: $d_i = 0.016$ m.

3 Calculation of ice slurry pressure drop

3.1 Calculation of pressure drops of ice slurry treated as a Bingham fluid

The analysis of flow curves and of the group of $c_f(\mathrm{Re}_B, \mathrm{He})$ curves shown in Fig. 4 indicates that the studied ice slurry can be treated as a Bingham fluid. For such fluids, the Hedström method [27] can be used in order to evaluate the pressure drops for laminar flow, and the Tomita method [20, 29] for the turbulent range. Using the Hedström method, the Fanning friction factor is determined from the following equation [27]:

$$\frac{1}{\mathrm{Re}_B} = \frac{c_f}{16} - \frac{\mathrm{He}}{6 \cdot \mathrm{Re}_B^2} + \frac{\mathrm{He}^4}{3 \cdot c_f^3 \mathrm{Re}_B^8} \tag{6}$$

and the pressure drops are calculated from equation (5).

In the turbulent region, the pressure drops for a Bingham fluid can be determined from the Tomita equations [20, 29] (7-10):

$$\frac{1}{\sqrt{c_{fBT}}} = 4.0 \cdot \lg\left(\mathrm{Re}_{BT}\sqrt{c_{fBT}}\right) - 0.4 \tag{7}$$

where

$$\mathrm{Re}_{BT} = \mathrm{Re}_B \frac{(1-\varepsilon_B)\cdot(\varepsilon_B^4 - 4\cdot\varepsilon_B + 3)}{3}; \tag{8}$$

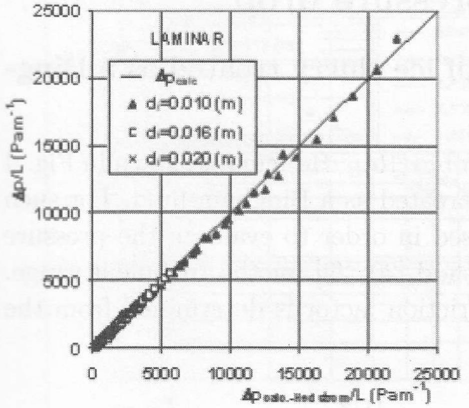
$$\varepsilon_B = \frac{\tau_p}{\tau_w} = \frac{4 \cdot L \cdot \tau_p}{d_i \cdot \Delta p}; \tag{9}$$

$$c_{fBT} = \frac{d_i \cdot \Delta p}{2 \cdot \rho_B \cdot L \cdot w_m^2 (1-\varepsilon_B)}. \tag{10}$$

The Hedström and Tomita methods have been verified for ice slurry flow by means of a comparison with the obtained experimental results. This is illustrated in Figs. 6 and 7 for all studied tube diameters. As can be seen, the Hedström method gives excellent agreement with the experiment, independent of the tube diameter. The Tomita method, used for turbulent flow, yields the values of pressure drops with an accuracy higher than 10%, for all diameters.

3.2 Generalised Reynolds number and generalised expressions for the Fanning friction factor

The generalised Reynolds number for non-Newtonian liquids permits to ascribe to them the pseudo-Newtonian liquid model. In literature, various definitions of



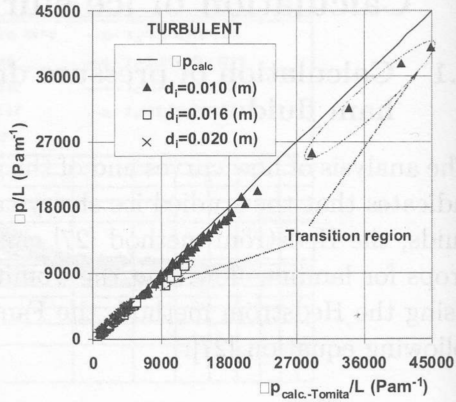


Figure 6. Comparison of experimental and calculated (based on the Hedström method) pressure drop values for a Bingham fluid, on the example of ice slurry.

Figure 7. Comparison of experimental and calculated (based on the Tomita method) pressure drop values for ice slurry.

the modified, generalised Reynolds number [11, 20, 22] are used. Compilation of some definitions is presented in Tab. 4.

For laminar flow independent of the Re_M definition, for all examined cases, the Fanning friction factor and Re_M obey the relation [20, 22, 27]:

$$c_{fM} = \frac{16}{\mathrm{Re}_M}. \tag{11}$$

In the turbulent flow region, the Fanning friction factor can be evaluated from

Table 4. Some definitions of the modified Reynolds number for a Bingham fluid.

Modified Reynolds number	Expression
Metzner-Read [22]	$\mathrm{Re}_{MR} = \dfrac{d_i^n w_m^{2-n} \rho}{K\, 8^{n-1}}$
Matras [20]	$\mathrm{Re}_{MM} = \dfrac{d_i w_m \rho_B \Gamma}{\tau_{wL}} \left[\dfrac{2\,(n+1))}{3n+1}\right]^{-2.5}$
Gücüyener - Mehmetolu [11]	$\mathrm{Re}_M = \mathrm{Re}_B \left(1 - \frac{4}{3}\varepsilon_B + \frac{1}{3}\varepsilon_B^4\right)$

the Blasius experimental relation [20, 27]:

$$c_{fM} = \frac{0.079}{\mathrm{Re}_M^{0.25}} . \tag{12}$$

In the case of ice slurry, for any definition of the generalised Reynolds number (Table 4), the experimental values of c_{fM} and Re_M exactly satisfy the Eqs. (11) and (12) (Fig. 8) [31].

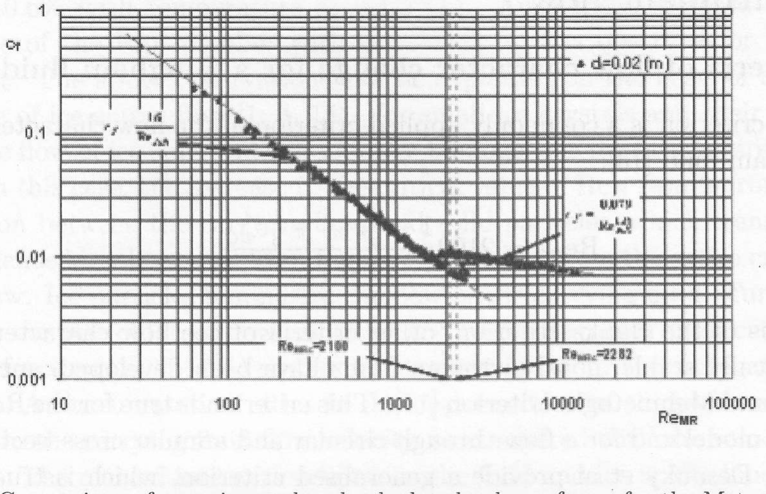


Figure 8. Comparison of experimental and calculated values of c_{fM} for the Metzner-Read generalised definition of the Reynolds number Re_{MR} for pipe diameter $d_i = 0.02$ m.

The different definition of the generalised Reynolds number Re_M we can order the different critical values of the number, at which the laminar–turbulent transition occurs. As an example, Fig. 8 presents a comparison of measured values of the Fanning friction factor with those calculated from Eqs. (11) and (12), using the Metzner – Read generalised definition of the Reynolds number [22]:

$$\mathrm{Re}_{MR} = \frac{d_i^n w_m^{2-n} \rho}{K\, 8^{n-1}} \tag{13}$$

where, for a Bingham fluid, the flow behaviour index n is determined using relation [27]:

$$n = \frac{1 - \frac{4}{3}\varepsilon_B + \frac{1}{3}\varepsilon_B^4}{1 - \varepsilon_B^4} . \tag{14}$$

The consistency index K is determined using relation [27]:

$$K = \tau_w^{1-n} \left(\frac{\mu_p}{1 - \frac{4}{3}\varepsilon_B + \frac{1}{3}\varepsilon_B^4} \right)^n . \tag{15}$$

Figures 6-7 show that for ice slurries one can use the methods of calculating pressure drops in the laminar and turbulent region, which are characteristic of a Bingham fluids. What is more, the experimentally determined Fanning friction factor values show a very good agreement with those obtained based on the generalised Reynolds number definition and Eqs. (11) and (12).

4 Criteria for the transition of ice slurry from laminar to turbulent flow

4.1 Criteria of flow character change for a Bingham fluid

The Hanks criterion is a commonly applied criterion of the flow character change for a Bingham fluid [13]:

$$\mathrm{Re}_{BC} = 2100\frac{1-\frac{4}{3}\varepsilon_{BC}+\frac{1}{3}\varepsilon_{BC}^4}{(1-\varepsilon_{BC})^3} . \tag{16}$$

On the basis of the Hanks criterion, other criteria of the flow character change for rheologically stable, non-Newtonian fluids have been developed, such as the Gücüyener and Mehmetoglu criterion [11]. This criterion is true for the Robertson – Stiff fluid model and for a flow through circular and annular cross–section duct [11]. In [5], Desouky et al provide a generalised criterion, which is true for the Oswald de Waele and Herschel–Bulkley plastic-viscous fluids model. In the case of a Bingham fluid, the above criteria correspond to the Hanks criterion.

The critical values of Re_{MRC} numbers at which laminar flow loses its stability, can also be evaluated from the Mishra–Tripathi relation. For a Bingham fluid, this relation has the following form [23]:

$$\mathrm{Re}_{MRC} = \frac{4200}{2-\varepsilon_{BC}} \tag{17}$$

The Maglione criterion [11] for Herschel–Bulkley fluids makes it possible to obtain reasonable values of the critical Re_{BC} numbers for large *He* numbers. From among the criteria cited in literature, for pseudo-Newtonian fluids the following criteria have been used: the Guillot criterion [11]:

$$\mathrm{Re}_{MRC} = 3250 - 1150n \tag{18}$$

and the Ryan-Johnson criterion [26]:

$$\mathrm{Re}_{MRC} = \frac{6464n}{(1+3n)^2}(2+n)^{\frac{2+n}{1+n}} . \tag{19}$$

4.2 The change of ice slurry flow character

The laminar – intermediate flow transition was observed within the range of $1800 < \mathrm{Re}_B < 3000$. It should be noted that the amplitude of pressure fluctuations observed when the flow was losing its stability was much higher for tube diameters $d_i \geq 0.016$ m than for $d_i = 0.01$ m. It was observed, that for ice content x_s <10% the critical value of the Reynolds number Re_B did not increase with increasing x_s, but it even fell slightly below the critical Re_B value for ethanol (x_{ai} = 10.6% with temperature at −4.5°C). An evident increase of the critical value of the Re_B number with increasing x_s was observed for ice content x_s > 10%. The above observations can be explained on the basis of the internal structure of ice slurry flow [1, 4, 15]. For small ice crystals and their larger contents, the flow of ice slurry was practically homogenous over the entire measuring range. In this case, the increase in the critical value of Re_B results from a mutual interaction between the carrying fluid and solid particles, which transfers a part of turbulence kinetic energy from the carrying fluid across the entire cross-section of the flow. Ice particles laminarize the flow of the carrying fluid. Turbulent flow occurs in the case of larger flow velocities and larger Reynolds numbers. A smaller ice mass fraction (x_s < 10%) and larger ice crystals favour the segregation of solid particles. An accumulation of ice in the upper section of the pipe results in the increase in the carrying fluid flow velocity, as compared with the velocity of solid particles. The mutual interaction between the fluid and ice is also hindered. This leads to an increase of carrying fluid turbulence and pressure drop, characteristic of turbulent flow. A change in the character of the mixture flow occurs when the flow velocity and Reynolds number values are lower.

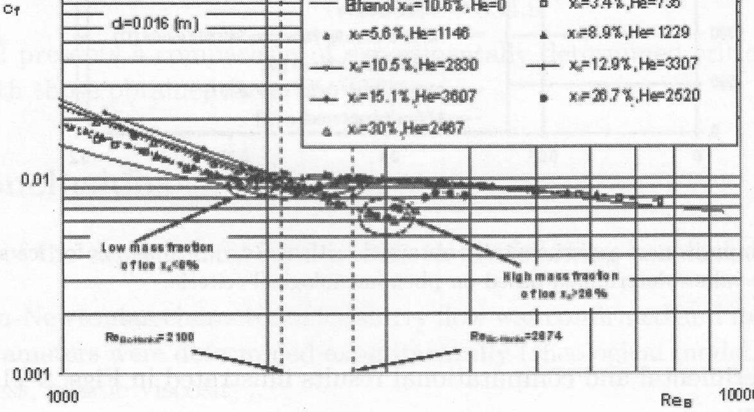


Figure 9. Variation of the critical Re_{BC} number with varying ice content for d_i = 0.016 m.

Figures 9 and 10 illustrate the variations of the critical Re_{BC} number with the change of ice content at different pipe diameters. A comparison of experimentally determined critical values of the Re_{MRC} numbers with the transition criteria cited above (within the experimental range of ε_{BC} variability) is presented in Fig. 11.

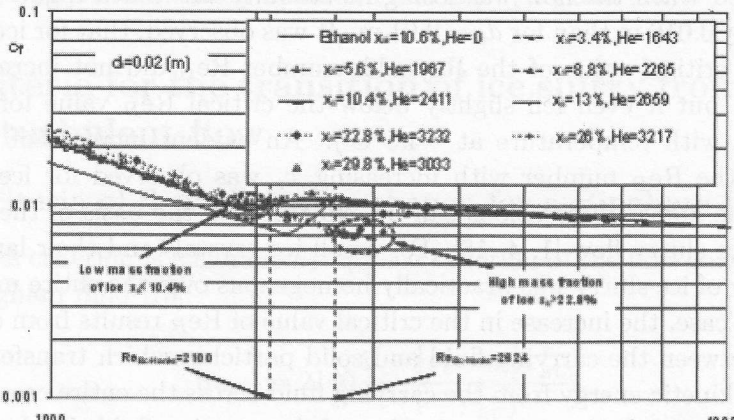


Figure 10. Variation of the critical Re_{BC} number with varying ice content for $d_i = 0.02$ m.

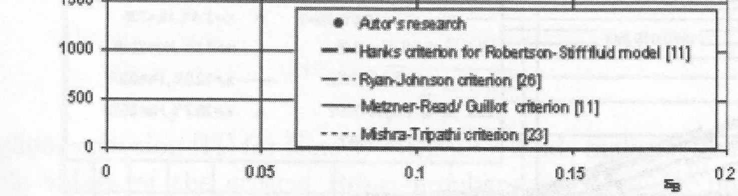


Figure 11. Comparison of experimentally obtained critical Re_{MRC} numbers for ice slurry with the values determined based on phenomenological criteria.

The experimental and computational results illustrated in Figs. 9-11 indicate the following:

- Among the criteria examined for ice slurry, the Hanks criterion permits to determine the critical values of Reynolds numbers with a maximum accuracy.

- For suspensions having a large mass fraction of solid particles ($x_s > 26\%$), the actual Re_{BC} numbers are larger than the maximum values resulting from the Hanks criterion, independent of the pipe diameter (Figs. 9-10).
- In other cases, the loss of stability of laminar flow occurs earlier than the Hanks criterion indicates(Fig. 1).
- For ice slurry with a small ice content (x_s <10%), there occurs a decrease of the critical Reynolds numbers below the critical values for the carrying fluid (Figs. 9-10).

It is important to notice that within the considered range of ε_B variability, the lowest critical value of the Reynolds number, determined on the basis of the above criteria, is equal to 2100. The absolute critical Reynolds number for suspensions is equal to 1200 [11, 20, 22].

The above observations indicate that for ice slurry the value of the critical Reynolds number is influenced not only by the physical and flow parameters of the suspension, but also by the parameters which determine the flow pattern [28]. Apart from the mean velocity, internal tube diameter, and the mass fraction of solid particles, another such parameter is the mean size of solid particles [15]. With the relative size of solid particles defined as d_s/d_i, an author's criterion of the laminar to turbulent transition was formulated. This criterion is true for the Hedström numbers He ≥ 200 and average ice crystal size of d_s=0.2mm. The critical value of the Reynolds number Re_{BC} can be determined from the following equation:

$$\mathrm{Re}_{BC} = \frac{10000\mathrm{He}\,(d_s/d_i)^{0.25}}{1.25\mathrm{He} + 334.9} \tag{20}$$

Figure 12 presents a comparison of experimentally determined critical values of Re_{BC} with those obtained from Eq. (18).

5 Conclusions

Analysis of the experimental results leads to the following conclusions:

- Non-Newtonian character of ice slurry flow was confirmed and its rheological parameters were determined experimentally (rheological model, yield shear stress, plastic viscosity).
- With an increasing fraction of solid particles, the loss of stability of the laminar flow takes place at higher flow velocity values.

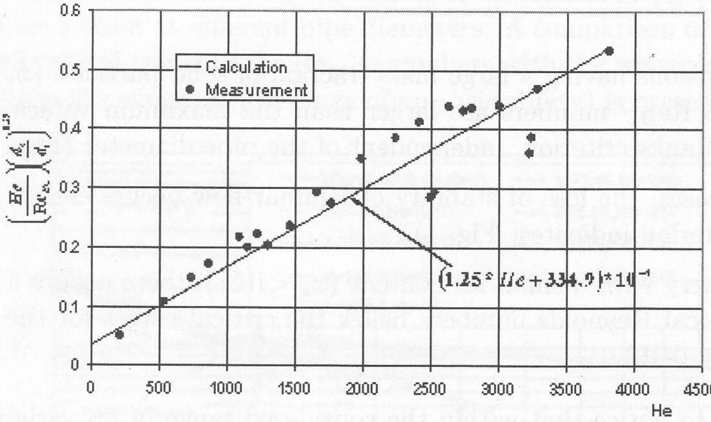


Figure 12. Comparison of experimental results with the author's criterion of laminar to turbulent transition for ice slurry.

- The $\Delta p/L$ versus x_s curves, describing the transition from laminar flow to unstable intermediate region, were determined experimentally. The observed intersection of the carrying fluid pressure drops curves and the $\Delta p/L$ versus w_m curves, for ice slurries with $x_S \geq 10\%$, could be explained. For some velocity ranges the carrying fluid flow is in the turbulent region while the flow of the fluid containing solid particles is still laminar. This is reflected by the values of the Reynolds numbers Re_B at which the flow loses its stability. In the case of the studied suspension this critical Reynolds numbers range is (1800 – 3000). Higher values correspond to higher fractions of solid particles. This effect was detected for three different values of the tube diameter. It can be referred to as "the laminarization of the flow", resulting from the fact that the solid particles take over a part of turbulence kinetic energy from the fluid.
- For transport purposes, it is recommended to use ice slurries with the highest possible fractions of solid particles (at least 20%) and maximum flow velocities, but with the mixture flow still in the laminar flow range.
- Determination of the Fanning friction factor as a function of Re and He showed that the maximum values of the Hedström number did not correspond to maximum values of x_s in the studied range $0 \leq x_s \leq 30\%$. The extreme values of the He numbers corresponded to ice contents of $15\% \leq x_s \leq 23\%$, independent of the tube diameter.
- In order to evaluate the pressure drops, it was possible to use the Hedström and Tomita methods related to Bingham plastics. In the stable laminar

and turbulent flow regions, the analytical methods were confirmed experimentally with a relative accuracy better than 10%.

- A comparison of the experiments with the simulation results and the Tomita model shows that, in all cases, the calculations give higher pressure drops. Relative differences between the empirical and calculated pressure drops are almost constant over the whole range studied in this work.
- When the generalised Reynolds number definition is assumed, the pressure drops of ice slurry flow over the laminar and turbulent range should be determined using the Fanning and the Blasius relation respectively. Mean relative accuracy between the measured and computed results is in this case lower than 10%.
- Mean values of the critical Reynolds numbers can be determined based on the Hanks criterion for Bingham fluids.
- An author's criterion of the change in the ice slurry flow character was proposed. The critical value of the Reynolds number was made dependent on the Hedström number and on the relative mean size of solid particles (d_s/d_i). Mean relative accuracy between the measured and computed results is in this case lower than 15%.

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