EIGENCHARACTERISTICS OF FLUID FILLED TANKS: AN EXTENDED SYMMETRICAL COUPLED APPROACH

ADAM WIŚNIEWSKI AND ROBERT KUCHARSKI

Institute of Fluid Flow Machinery, Polish Academy of Sciences,
Fiszera 14, 80-952 Gdansk, Poland
robertk@imp.gda.pl

(Received 2 September 2006; revised manuscript received 26 September 2006)

Abstract: In the numerical analysis of the eigen behaviour of large liquid and gas storage tanks, an important role is played by initial pre-stressing of such thin-walled structures due to high fluid pressure and gravity. In a majority of numerical simulations, the finite state of deformation is first calculated, following which small, linear vibrations are superimposed on the finite state. This paper is devoted to refinement and assessment of the basic Eckart superimposed eigencharacteristics problem simultaneously stated in fluid and thin-walled structures. Eckart's coupled approach leads to variational structure-fluid coupling in the so-called acoustic approximation. In order to verify the feasibility and correctness of the symmetrical Eckart approach, finite element discretization and a calculation example of a rectangular tank are presented. The calculated results are compared with literature ones.

Keywords: fluid-structure interaction, free-vibration, modal analysis, acoustic method, FEM

1. Introduction

Rectangular gas and liquid storage tanks are the main elements of many types of engineering structures, ind. marine, aerospace, petrochemical, nuclear and power generating, so their dynamic and vibration problems are important for the design and operation of such structures. Therefore, a lot of research has been devoted to the eigencharacteristics problem [1–4]. A separate fluid-solid modal analysis technique called mass-adding has been widely discussed in the literature as means to solve this problem [3, 5]. However, a coupled fluid-structure vibration analysis approach has been developed recently [2–4, 6, 7], mainly based on the so-called acoustic approximation, limited to fluid media. In the present paper we attempt to extend a similar approach to a solid medium. This offers an opportunity to analyze a tank which has been initially stressed, deformed due to stationary creep, *etc.* A coupled structure-acoustic formulation is well-known to require symmetrization, as it adequaly depends on which acoustic primary variable of the fluid has been taken into consideration.

The formulation of Zienkiewicz and Bettess [6] has been based on acoustic pressure, while Chen [4] and Everstine [8] have considered the velocity potential. However, both formulations are inherently bound to asymmetric or complex matrix equations [9]. Everstine has suggested introducing the acoustic velocity potential, in addition to sound pressure, as an independent variable in the fluid domain [8]. The effective coupled symmetric formulation, based on mixed acoustic pressure and displacement potentials has been introduced by Morland and Ohayon [2] and Sandberg and Goransson [10]. Nevertheless, the latter model cannot take excitation of the acoustic source into account as conservation equations of mass and momentum in the fluid domain have been introduced into their deducing course [7]. Extending the above formulations, Chen and Taylor [11] used finite elements based on a kind of displacement vectors in solid and fluid.

In this paper, starting from the Chen and Taylor approach, we intend to introduce over-displacement vectors in solid and fluid media which describe small superimposed motion on the finite state of the pre-stressed fluid-solid continuum. Such superimposed small displacement is well-known in structural analysis of vibrational acoustics and stability, however in this kind of fluid displacement was first proposed and analytically motivated by Eckart [12]. The advantage of Eckart's approach is that small motion can be superimposed on an arbitrary finite state of fluid – *e.g.* following, turbulent, under pressure and in phase transition. In spite of remarkable advances in techniques of numerical analysis for fluid-structure interaction considering the symmetry of superimposed small displacements, a test program is still needed for coupled free-vibration study of fluid-filled tanks. In the present paper, coupled free-vibration analysis has been examined and tested using our implementations into the current commercial Finite Element Method software. The theoretical work and the test program have been verified by comparison with the available numerical solutions. The parametric study of the natural frequency has been examined in various combinations of pre-stressed state of thin-walled structures and fluids.

Yet another problem is connected with fully symmetrical coupling in the structure-fluid formulation, stamming from different mathematical modelling of the motion of thin-walled, shell-like structures and that of fluid motion. The incompatibility of traditional formulations is due to the difference between two-dimensional shell modelling and three-dimensional fluid modelling. In numerous formulations of the coupled symmetric structure-fluid problem, the free-vibration problem has been approximated with degenerated 8 or 6-node 2D shell elements and 3D fluid elements [7, 13]. Degenerated shell elements always require the application of the reduced integration technique to prevent the shear and membrane locking in statics [14–16] and similar mass-locking effects in free-vibration [17]. In the case of fluid finite elements, reduced integration is also required for elements of low approximation[5]. At the same time, it follows from previous studies [15, 18, 19], that every pre-stressed background finite state minimalizes the effect of reduced integration. Therefore, in the present paper, 3D finite elements will be tested simultaneously for the solid and the fluid medium and the effect of reduced integration will be verified. This approach has an additional advantage of facilitating discretization of complicated fluid partially-filled structures with several substructures of arbitrary shape. Additionally, in such 3D modelling there is no complexity connected with special treatment of two rotation components of the mid-surface and a rotation component of the shell's mid-surface [19].

2. Eckart's approach to symmetrical structure-fluid superimposed motions

Let us assume that a thin-walled tank filled with fluid in an arbitrary state can, in general, undergo finite state deformation (elastic, creep, thermal, *etc.*). Initial deformation is mainly due to from internal actions of the fluid: pressure of gas or gravity of liquid. Another initial load is due to forces of inertia, centrifugal forces and external loads, point supports, *etc.* Initially pre-stressed and deformed structures are characterized by an additional stiffness and much higher frequencies than stress- and deformation-free structures. Quite the same can be said about elastic fluids – the influence of pressure and gravity forces can be observed in the so-called acoustic characteristics of fluids. In the problem of modelling small perturbations superimposed on a finite state of a solid/fluid continuum, it is assumed that the perturbations will propagate as small amplitude elastic waves. Therefore, even if the finite state consists of irreversible contributions, *e.g.* plastic deformations, viscous stresses and turbulent losses, we omit any viscous and irreversible contributions when modelling the superimposed motion. Similarly, small superimposed small perturbations such as vibrational displacements or density and entropy fluctuations shall treated as purely elastic. Then, the recoverable energy of superimposed motion in solid-fluid coupled medium is defined as the difference between internal, kinetic and potential energies [2, 3]:

$$L = L_{\text{fluid}} + L_{\text{solid}} + L_{\text{coupling}}, \tag{1}$$

where:

$$L_{\text{fluid}} = \iiint\limits_{V_f} \rho_f (\varepsilon_f - \frac{1}{2} g^f_{ij} v_i v_j + V(\vec{x})) dV_f + D, \tag{2}$$

$$L_{\text{solid}} = \iiint\limits_{V_s} \rho_s (\varepsilon_s - \frac{1}{2} g^s_{ij} \dot{u}_i \dot{u}_j + V(\vec{x})) dV_s, \tag{3}$$

$$L_{\text{coupling}} = \iiint\limits_{\text{surface}} p_f u_i n_i dA. \tag{4}$$

In the above relations ρ_s and ρ_f respectively denote density of the solid and the fluid, $\vec{v} = v_i \vec{e}_i = \dot{\vec{u}}$ – the superimposed fluid velocity, $\vec{u} = u_i \vec{e}_i$ – the superimposed displacement of the solid, ε_f and ε_s – the solid's and fluid's specific densities of internal energy in superimposed motion, $V(\vec{x}) = zg$ – a specific body force potential at point $\vec{x} \in V_f$ or $\vec{x} \in V_s$, p_f – the superposed fluid pressure on the surface of a solid body, and $\vec{n} = n_i \vec{e}_i$ – a normal vector orienting surface of ∂V_s. Finally, D is the dissipation functional.

Calculating kinetic energy of small perturbations superimposed on a finite state is not an obvious problem and, in some cases, cannot be reduced to half a square of velocity, *i.e.* $\frac{1}{2} v_i v_i$. In his analysis of acoustic vibration, Eckart [12] introduced the so-called kinetic metric of acoustic fluid, g^f_{ij}. Its mathematical interpretation is a symmetric metric tensor that describes the dynamics of a fluid in a finite state. In the classical formulation of kinetic energy one has $g_{ij}^{f} = \delta_{ij}$, what physically has the same meaning as, for instance, the kinetic energy of a stone thrown into an empty space. Following Eckart, we have to introduce a kinetic metric of a solid body, g_{ij}^{s}. Both objects, g_{ij}^{f} and g_{ij}^{s}, are symmetrical, positively defined covariant metrics which are entirely determined as a function of parameters of a finite, pre-stressed state. Following the tradition, w shall denote these parameters by additional index, $(.)^0$. These are parameters like the state parameters: p^0, T^0, v^0, s^0 in a fluid medium and σ_{ij}^0, T_{ij}^0, ε_{ij}^0, s_{ij}^0 in a solid medium.

From the solid body point of view, L_{coupling} can be treated as the work of external loading [4], due to a known fluctuation pressure, p_f. In the case of a still fluid on which small perturbations are superimposed, p_f is identical to the fluctuations of hydrostatic pressure (for a liquid) or fluctuations of gas pressure in a pre-stressed tank. The specific internal energy of small perturbations of the finite state is described as a small linear increment of internal energy defined as:

$$\varepsilon_s = \frac{1}{2}\varepsilon_{ij}A_{ijkm}^0\varepsilon_{km} + \varepsilon_{ij}B_{ijkm}^0 s_{km} + \frac{1}{2}s_{km}C_{ijkm}^0 s_{km} \quad [\text{J/kg}], \tag{5}$$

$$\varepsilon_s = \frac{1}{2}\theta A^0\theta + \theta B^0 s + \frac{1}{2}sC^0 s \quad [\text{J/kg}]. \tag{6}$$

As all coefficients of the A, B, C type in the above definitions are calculated on the finite pre-stressed state, they are referred to as acoustic tensors and coefficients of elasticity. With Kirchhoff's assumption that temperature and entropy tensors in solids are nearly spherical, the part with the B_{ijkm}^0 coefficient in Equation (5) can be approximated to:

$$\varepsilon_{ij}B_{ij}^0(s_1 - s_0) \tag{7}$$

and the acoustical tensor of the entropy constant, C_{ijkm}^0, becomes a scalar, C^0. In the above relations $\varepsilon_{ij} = 1/2(u_{i,j} + u_{j,i})$ is a tensor of small deformations defined on the Cartesian metric as the metric g_{ij}^{s} is meaningful only for kinetic energy. Here, $\vec{u} = u_i\vec{e}_i$ is a vector of small perturbations of the displacement field which is taken from the pre-stressed state, s_{ij} is an increment of the entropy tensor (in solids only), $\theta = \operatorname{div}\vec{u} = \varepsilon_{ii}$ is a small volumetric deformation produced be small perturbation and s is a volumetric increment of entropy of the perturbed fluid.

The A^0 coefficient in Equation (6) should be interpreted as a coefficient of isenthalpic volumetric elasticity, which allows us to express small perturbated pressure as follows:

$$p_f = \frac{\partial\varepsilon_f}{\partial\theta}|_{s=\text{const}} = A^0\theta. \tag{8}$$

It is assumed here that a state of small perturbations superimposed on a fully non-linear, finite state of fluid in motion is further properly described by elastic ideal fluid model that cannot include any shear deformation. It practically means that the general form of Equation (5) including anisotropy induced by the finite state, now takes the form of a classical Navier-Lamé constitutive relation describing linear relation between stress and deformation in an isotropic medium:

$$\sigma_{ij} = A_{ijkm}^0\varepsilon_{km} + B^0\delta_{ij}s = \lambda^0\theta\delta_{ij} + 2\mu^0\varepsilon_{ij} + B^0\delta_{ij}s, \tag{9}$$

which when fluid perturbations are transported only in the volumetric manner ($\mu^0 \equiv 0$), leads to:

$$\sigma_{ij} = p\delta_{ij} = \left(\lambda^0\theta + B^0 s\right)\delta_{ij}. \tag{10}$$

By comparison with Equation (8) the acoustic coefficient of volumetric elasticity is found to be equal to $A^0 = \lambda^0$, like the first Lame's coefficient in a solid medium. We shall call it the acoustic model of volumetric elasticity. It appears in the constitutive equations of liquids and gasses connecting fluctuating pressure, p_f, with volumetric fluid deformation, $\theta = \operatorname{div}\vec{u} = \varepsilon_{ii}$. It is also known as a volumetric form of the linear Hooke's equation. If increases of pressure, Δp, and increases of volume are known from experiment, the λ^0 modulus may be obtained from the following equation:

$$\lambda^0 = -\frac{\Delta p}{\Delta V/V}. \tag{11}$$

Under the action of pressure increase Δp, any initial liquid volume V decreases its volume by ΔV. As fluids cannot accumulate elastic shear deformations, the lateral Lamé module will always equal zero, $\mu^0 = 0$. However, it is assumed for an ideal gas [3] that the acoustic module of volumetric elasticity is equal to the actual gas pressure in the finite state:

$$\lambda^0 = p^0. \tag{12}$$

For atmospheric air, the value of coefficients taken for numerical analysis have been as follows: elasticity modulus $\lambda^0 = 0.1\,\mathrm{MPa}$, density $\rho = 1.27\,\mathrm{kg/m^3}$. However, the acoustic modulus of volumetric elasticity for any working liquid that contains an amount of the gaseous phase may vary significantly. If α denotes the volumetric gas fraction dispersed in a liquid, then Equation (11) should be replaced by the following [13]:

$$\lambda_m^0 = \frac{\lambda_c^0}{1 + \alpha(\lambda_c^0/\lambda_g - 1)}, \tag{13}$$

where $V_g/V = \alpha$ is the volumetric fraction of gas in the mixture, index m meaning mixture, c – liquid, g – gas. For pure water at atmospheric pressure and in ambient temperature the acoustical modulus of volumetric elasticity has been taken as equal to $\lambda^0 = 200\,\mathrm{MPa}$, with density $\rho = 1000\,\mathrm{kg/m^3}$. It should be noted, that there is a lot of diluted air in an actual liquid contained in a tank. Therefore, the acoustical modulus of volumetric elasticity, even if calculated from Equation (13) is practically known with some exactness, since parameter α can change drastically in operation. The analytical form of acoustical tensor A_{ijkm}^0 has been discussed in the literature [1, 15]. Actually, many commercial codes, in order to calculate the eigenvalue problem for a structure undergoing finite state of stresses and deformation, must calculate the tensor numerically.

3. Variational and finite element formulation

The energetical formulation presented above is appropriate for solving the problem of small perturbations propagating in a coupled fluid-solid system. Additionally, the energetical formulation leads directly to the matrix approach used in the finite element method. Using the Hamiltonian principle (see [2, 19]) which asserts that the difference between kinetic and potential energy is constant during virtual motion taken between arbitrary temporial moments t^n and t^{n+1} we obtain:

$$\delta H = \delta \int_{t^n}^{t^{n+1}} L\,dt \equiv 0. \tag{14}$$

With space discretization typical for the finite element method, the fluid and structural domains can be divided into elements. Expanding wave displacement and wave entropy using the following classical shape functions:

$$u_i = \boldsymbol{N}_i \boldsymbol{q}, \quad s = \boldsymbol{N}_s \boldsymbol{q}, \quad i = x, y, z, \tag{15}$$

a set of finite element formulae can be deduced in a relatively straightforward way:

$$\boldsymbol{M}^0 \ddot{\boldsymbol{q}} + \boldsymbol{C}^0 \dot{\boldsymbol{q}} + \boldsymbol{K}^0 \boldsymbol{q} = \boldsymbol{F}^0. \tag{16}$$

It is a discrete form of the perturbed state, where particular matrices are interpreted as mass, viscosity and acoustic stiffness matrices, while $\boldsymbol{F}^0$ is the perturbed forces vector:

$$\boldsymbol{M}^0 = \iiint_{V_\alpha} \boldsymbol{N}_i^T g_{ij}^{\alpha} \boldsymbol{N}_j dV_\alpha, \quad \alpha = f, s, \tag{17}$$

$$\boldsymbol{C}^0 = \iiint_{V_f} \boldsymbol{N}_i^T \boldsymbol{D}_{ikl}^T D_{klmn}^0 \boldsymbol{D}_{jmn} \boldsymbol{N}_j dV_f, \tag{18}$$

$$\begin{aligned}
\boldsymbol{K}^0 = & \iiint_{Vs} \boldsymbol{N}_i^T \boldsymbol{D}_{ikl}^T A_{klmn}^0 \boldsymbol{D}_{jmn} \boldsymbol{N}_j dV_s \\
& + \iiint_{V_s} (\boldsymbol{N}_i^T \boldsymbol{D}_{ikl}^T B_{kl}^0 \boldsymbol{N}_s + \boldsymbol{N}_s^T B_{mn}^0 \boldsymbol{D}_{jmn} \boldsymbol{N}_j) dV_s \\
& + \iiint_{V_s} (\boldsymbol{N} s^T C^0 \boldsymbol{N} s) dV_s \\
& + \iiint_{V_f} \boldsymbol{N}_i^T \boldsymbol{D}_{i\theta}^T A^0 \boldsymbol{D}_{j\theta} \boldsymbol{N}_j dV_f \\
& + \iiint_{V_f} (\boldsymbol{N}_i^T \boldsymbol{D}_{i\theta}^T B^0 \boldsymbol{N}_s + \boldsymbol{N}_s B^0 \boldsymbol{D}_{j\theta} \boldsymbol{N}_j) dV_f \\
& + \iiint_{V_f} (\boldsymbol{N}_s^T C^0 \boldsymbol{N}_s) dV_f \\
& + \iint_{\text{surf.}} (\boldsymbol{N}_p^T \boldsymbol{D}_p \boldsymbol{D}_j \boldsymbol{N}_j + \boldsymbol{N}_j^T \boldsymbol{D}_j \boldsymbol{D}_p \boldsymbol{N}_p) dA.
\end{aligned} \tag{19}$$

Let us note that in the above definition the form of mass matrix is set up by a formula identical for solid and fluid domains. Matrices of the $\boldsymbol{D}$ type, which have appeared in Equations (17)–(19), contain spatial derivatives of an acoustic displacement vector, and acoustic constants of the A^0, B^0, C^0 type are determined by Equations (5)–(6). We shall now consider a motionless fluid and assume the perturbation process to be isentropic ($s = 0$), obtaining a simpler formula.

4. Problem solution

If no external loads act on the fluid-structure system, then $\boldsymbol{F}^0(t)=0$ in Equation (16) and one obtains a matrix equation describing damped free perturbations:

$$\boldsymbol{M}^0\ddot{\boldsymbol{q}}(t)+\boldsymbol{C}^0\dot{\boldsymbol{q}}(t)+\boldsymbol{K}^0\boldsymbol{q}(t)=\boldsymbol{0}, \tag{20}$$

which embrace simultaneously the solid and the fluid domains. The frequencies of the system's acoustic vibration are called eigenvalue frequencies. The solution of the above equations independently for the solid or the fluid, assumes the following form of $\boldsymbol{q}=\boldsymbol{q}_a e^{\lambda t}$ where $\boldsymbol{q}_a$ and λ are complex constants. Combining algebraic Equation (20) with the harmonic motion assumptions we arrive at a free-vibration matrix system for fluid-structure problems:

$$(\boldsymbol{M}^0\lambda^2+\boldsymbol{C}^0\lambda+\boldsymbol{K}^0)\boldsymbol{q}_a=\boldsymbol{0}. \tag{21}$$

When vibrations occur in pre-stressed solid and fluid medium, the symmetric tangent mass, damping and stiffness matrices in Equation (21) differ completely from the well-known linear matrices of mass, damping and stiffness, $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ [9]. For uncoupled free vibration the damping matrix equals zero and the problem leads to:

$$\boldsymbol{M}^0\ddot{\boldsymbol{q}}(t)+\boldsymbol{K}^0\boldsymbol{q}(t)=\boldsymbol{0}, \tag{22}$$

with the solution defined by $\boldsymbol{q}=\boldsymbol{q_a}\sin(\omega t)$, where $\boldsymbol{q}_a$ is a vector of eigenvalue form, ω – frequency [rad/s], $f=\omega/2\pi$ – frequency [Hz] and $T=1/f$ – period [s]. After double differentiation one obtains linearized free vibration for the structure-fluid coupled problem in a symmetrical formulation. The problem describes small perturbations (density, displacements and entropy) superimposed on the initial finite state of stresses and deformations:

$$\left(\boldsymbol{K}^0-\omega^2\boldsymbol{M}^0\right)\boldsymbol{q}_a=\boldsymbol{0}. \tag{23}$$

Owing to the symmetry, this matrix system does not require expensive computation and needs to be solved as the following classical eigenvalue problem:

$$\det\left|\boldsymbol{K}^0-\omega^2\boldsymbol{M}^0\right|=\boldsymbol{0} \tag{24}$$

The solution of Equation (24) is to be found numerically using standard procedures developed in the Abaqus and Nastran commercial codes. It is known from algebra that under the positive definite condition of matrix $\boldsymbol{M}$ all eignevalues are positive if matrix $\boldsymbol{K}$ is positive definite and non-negative if matrix $\boldsymbol{K}$ is positive semi-definite [20]. In general, matrixes $\boldsymbol{M}$ and $\boldsymbol{K}$ in Equation (24) do not satisfy the above conditions and so non-negativity conditions of the system's eigenvalues must be investigated. For empty tanks, zero eigenvalues correspond to zero fluid pressure or constant acoustic pressure with zero initial entropy.

5. The subject of numerical analysis

In order to verify the above presented model we have performed a parametrical analysis of a rectangular fluid filled tank 1m high and wide, and 3m long. All the wallboards consisted of steel plates 1cm thick. The tank was clamped at the bottom to a rigid foundation. The material data of the steel were as follows: Young's modulus E=21 000MPa, Poisson ratio $v=0.3$, density $\rho=7800\,\mathrm{kg/m^3}$.

In order to verify the differences in eigenfrequencies for various filling fluids, the following four cases were investigated numerically:

(1) an empty tank,
(2) an air-filled tank with various overpressures,
(3) a water-filled tank under atmospheric pressure, without the forces of gravity,
(4) a water-filled tank with hydrostatic pressure.

The first case, which corresponds to the classical eigenvalue problem for an initially free structure (no pre-stressing or initial deformations), was used as reference data. It is especially important in checking the correctness of the finite elements, which, in some situations, may be indicative of the locking phenomenon. Therefore, solutions based on 3D solid elements should be compared with those based on the more consistent 2D shell elements that do not tend to lock in static and dynamic analysis [15, 17, 18].

The second case was the same tank filled with air compressed under various pressures. It should indicate how the eigenvalues of tanks increase with increasing air pressure from 0.1MPa to 20MPa. In this case, the initial deformation and initial stress state at the solid structure was neglected due to small air pressure and analysis was focused on proper modelling of the acoustic bulk elastic modulus of the compressible, isotropic fluid. The third case, with air replaced with nearly incompressible water was quite similar.

6. The FEM model in space

The numerical model contains two types of documents from library of FEA codes: MSC Nastran and HKS Abaqus. Two types of elements were used in the 2D and the 3D model, respectively. 3D elements are often sensitive to locking problems [1].

Two ways of modelling cases were tested:

- the tank walls were modelled with 8-node plate elements while the fluid was modelled with standard 20-node solid 3D element, see Figure 1a)
- the solid and the fluid were modelled with 20-node solid 3D elements, see Figure 1b).

Model (a) shown in Figure 1 contains 3751 nodes. 1402 nodes describe the tank walls. Model contains 9851 degrees of freedom, with 341 nodes (682 degrees of freedom) fixed to the tank bottom.

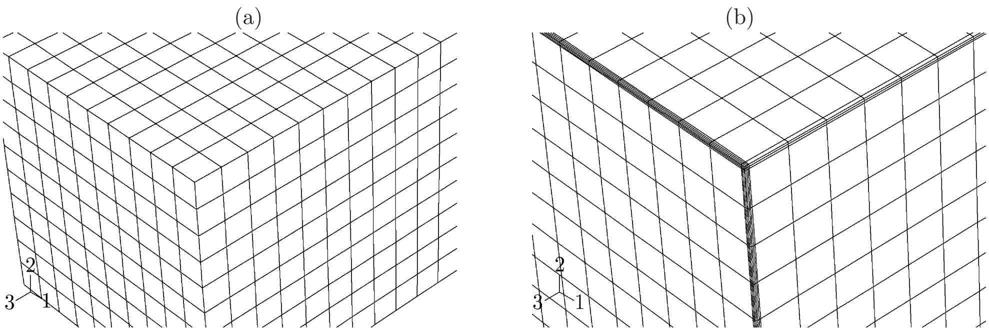

Figure 1. Tank model: (a) 3D elements (fluid) & 2D elements (solid); (b) 3D elements (solid & fluid)

Model (b), with the fluid and the tank walls modelled with 20-node elements contains 26 264 nodes (of which 18 072 fluid). In this paper we have also compared results obtained with 3D 8-node elements. In model containing 8-node elements the number of nodes was 1054.

7. Numerical analysis of the virtual stiffness influence

7.1. Natural frequencies of empty and air-filled tank

Natural frequencies of the empty and air-filled tank are presented in Table 1. In the latter case the pressure of air was about 0.1MPa.

Table 1. Natural frequencies of empty and air filled steel tank at $p = 0.1\text{MPa}$

	Empty tank		Filled tank	
Mode	Shell 8-node element	Solid 20-node element	Shell 8-node element	Solid 20-node element
1	35.55	35.07	36.41	35.97
2	43.89	43.78	44.67	44.60
3	49.62	48.98	50.14	49.55
4	56.44	56.24	57.00	56.84
5	57.81	58.19	58.49	58.91
6	59.41	59.02	60.15	59.80
7	63.79	63.81	64.46	64.52
8	68.09	68.51	68.65	69.10
9	69.72	70.12	70.30	70.73
10	74.17	74.75	74.75	75.37

It follows from the results presented in Table 1 that the three dimensional elements does not show the locking effect and yields results which are comparable with those from the shell elements. The influence of air is very small. The differences between eigenvalues are less than 1%. It is a result of small acoustic stiffnes of air. In Section 7.4 below the dependence of gas pressure and the acoustic stiffness factor is presented, so in calculations of gas-filled tanks at atmospheric pressure modelling of fluid can be neglected. Calculations for highly stressed tank walls have shown that locking of 3D elements is small. Similar results are presented in papers [1] and [2], only linear states without high loads are susceptible to locking.

7.2. Frequencies of the tank filled with water

Natural frequencies of the water-filled tank were calculated by the second way of modelling (3D elements only). The results are presented in Table 2. Two types of elements were used, 8-node brick elements and face-centered second-order 20-node elements. Calculations were performed for the following fluid parameters:

- acoustic modulus $\lambda^0 = 200\text{MPa}$,
- density $\rho_f = 1000$ kg/m^3.

The twenty-node elements yielded results similar to those of the shell elements. Eight-node elements were subject to locking, resulting from low-level shape functions. In dynamic calculations and/or eigenvalue extractions it is advisable to use at least second-order elements.

Table 2. Natural frequencies of the water-filled tank in Hz

	Nastran		Abaqus	
Mode	2D element 8 nodes	3D element 20 nodes	2D element 8 nodes	3D element 20 nodes
1	106.08	106.25	105.96	113.87
2	161.78	163.67	162.85	178.96
3	175.23	176.95	176.78	193.70
4	188.84	189.68	188.32	207.80
5	193.24	195.86	195.11	213.38
6	197.64	200.84	199.87	222.69
7	210.53	213.86	212.34	235.00
8	218.87	223.27	221.86	245.85
9	227.02	230.95	229.52	250.73
10	228.26	231.87	230.54	263.54

7.3. Comparison with air

Natural frequencies of the air- and water-filled tank are presented in Table 3. Natural frequencies of the water-filled tank are higher than when filled with air, since a fully filled tank vibrates like a solid beam. In an air-filled tank every wall vibrates independently, as can bee seen in Figures 2–4. The empty tank modes have been omitted as they are identical with those obtained for the air-filled tank at atmospheric pressure.

Table 3. Natural frequencies of the air- and water-filled tank

	Air-filled tank		Water-filled tank	
Mode	8-node (2D elements)	20-node (3D elements)	8-node (2D elements)	20-node (3D elements)
1	36.41	35.97	106.08	106.25
2	44.67	44.60	161.78	163.67
3	50.14	49.55	175.23	176.95
4	57.00	56.84	188.84	189.68
5	58.49	58.91	193.24	195.86
6	60.15	59.80	197.64	200.84
7	64.46	64.52	210.53	213.86
8	68.65	69.10	218.87	223.27
9	70.30	70.73	227.02	230.95
10	74.75	75.37	228.26	231.87

The result have shown that in this case fluid modelling can be neglected. As a result of modelling the internal fluid its mass is added to the global mass matrix and its acoustic stiffness to the global stiffness matrix, so that the fluid's influence and interactions of the tank walls with the fluid can be archived.

7.4. The first natural frequency versus λ^0 (calculated for water)

The acoustic elastic modulus of water is a free model parameter that requires calibration. In order to present the influence of λ^0 on the tank's vibrations, we have carried out calculations in the high range of λ^0 values, $\lambda^0 \in [50,1000]$MPa. The calculations included the influence of stresses from hydrostatic pressure. Table 4 presents the dependence of the first natural frequency on λ^0.

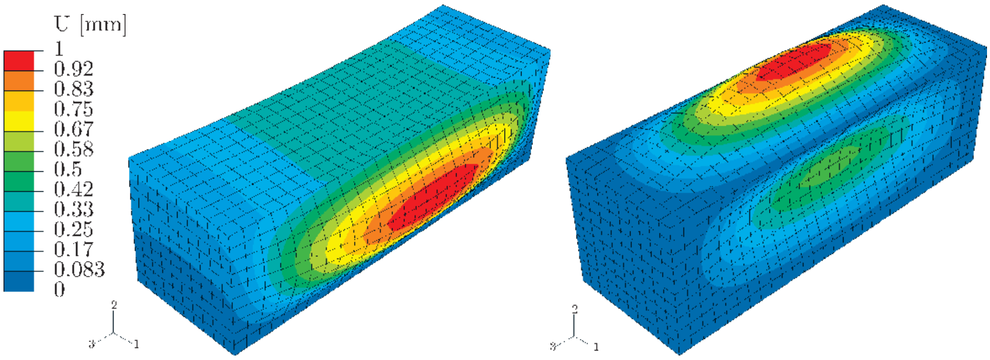

Figure 2. The first natural frequency of the water-filled tank (left, 106.08 Hz) and the air-filled one (right, 36.41 Hz)

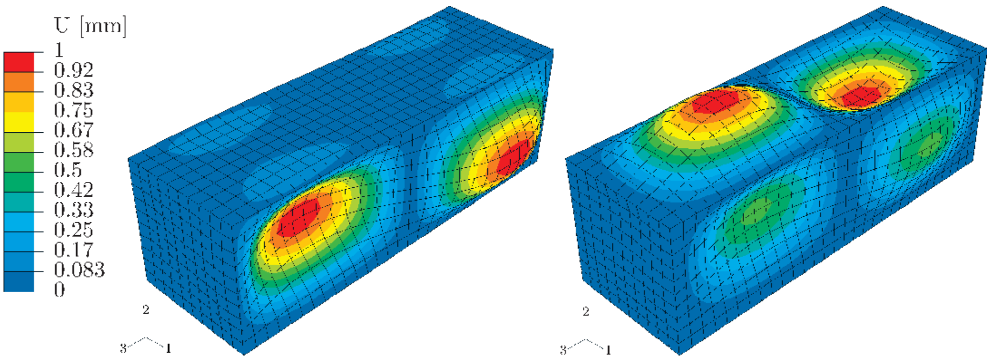

Figure 3. The second natural frequency of the water-filled tank (left, 161.78 Hz) and the air-filled one (right, 44.76 Hz)

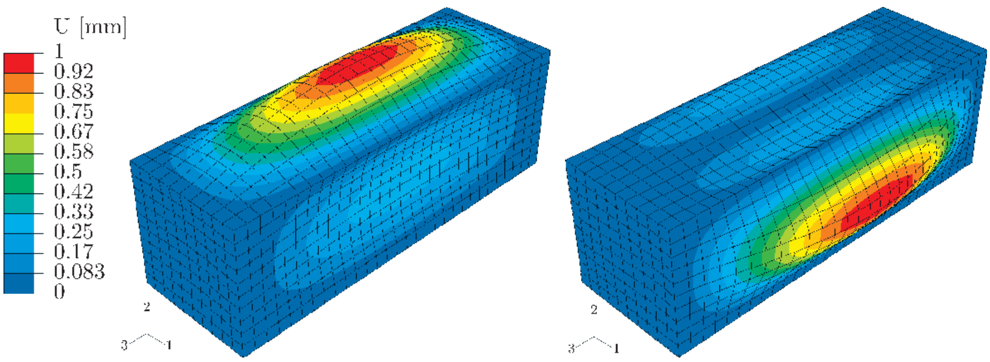

Figure 4. The third natural frequency of the water-filled tank (left, 175.23 Hz) and the air-filled one (right, 50.14 Hz)

Table 4. The first natural frequency versus λ^0, parameters for tap water in boldface

Acoustic modulus λ^0 [MPa]	50	100	**200**	400	700	1000
First natural frequency [Hz]	70.4	87.3	**108.0**	129.6	154.96	174.96

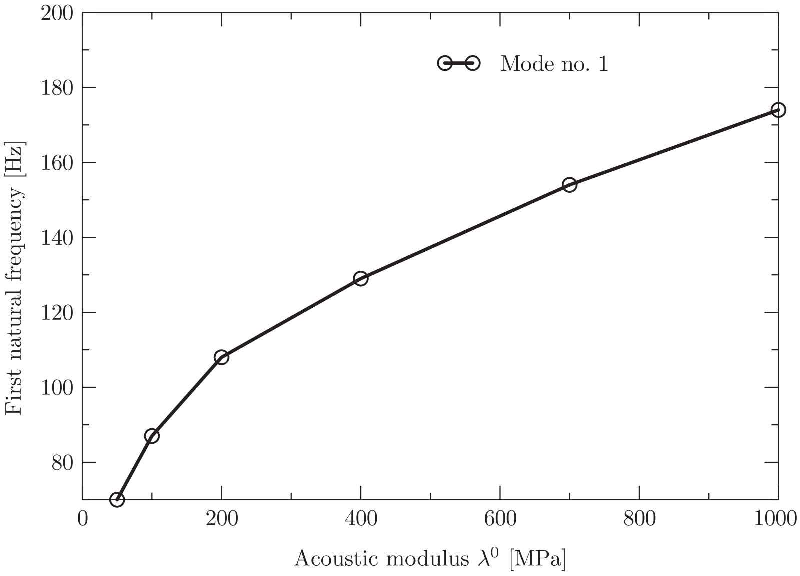

Figure 5. The first natural frequency versus λ^0 calculated for the water-filled tank

A strong dependence of the first natural frequency on λ^0 is noticable in Figure 5. A small change of λ^0 leads to large changes in the first natural frequency values.

7.5. The first natural frequency of the tank versus air pressure

For air (gas) λ^0 modulus equals pressure. Calculation data and the first natural frequency of the air-filled tank at various pressures are presented in Table 5 and in Figure 6. The value of the first natural frequency increases with increasing pressure.

Table 5. The first natural frequency versus λ^0 for the air-filled tank

Pressure [MPa] $\equiv \lambda^0$	Density [kg/m^3] at 20°C	The first natural frequency [Hz]
0.1	1.196	36.42
0.3	3.558	38.08
0.5	5.938	39.66
0.7	8.318	41.16
1.0	11.892	43.28
1.5	17.853	46.52
2.0	23.822	49.47
5.0	59.8	63.05
10.0	120.42	76.53

When air pressure is very high, the density of air increases to the value of water and the natural frequencies of the air-filled tank are close to those obtained for the water-filled tank.

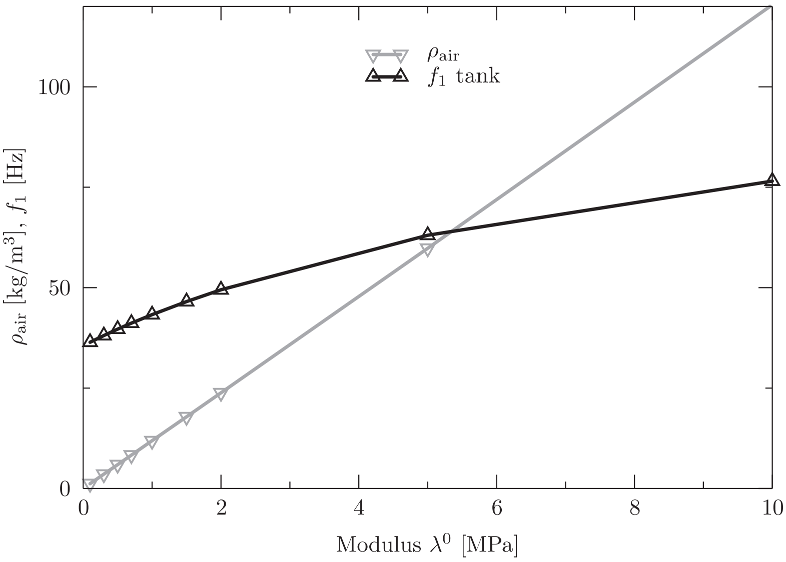

Figure 6. The first natural frequency versus λ^0 for air

8. Conclusions

The performed calculations have proven that the common fluid-and-solid modelling technique is an effective way of calculating compressible and non-compressible fluid-filled structures. Only liquid-filled structures require calibration of the acoustic λ^0 modulus, which equals 200MPa for tap water. Calculating the C^0 matrix connected with molecular viscosity of liquid remains an unsolved problem. The presented method, in comparison with the standard "added mass method" enables full modelling not only of the mass matrix but also of the stiffness matrix. Its cost is only an increased number of degrees of freedom of the calculated structure.

The results presented in this paper have demonstrated that locking phenomena can be neglected when calculations are performed with second order elements or when the structure is loaded. 8-node brick elements have been shown to be subject to the locking effect even in the low-stressed state so cannot be used in dynamic analysis or eigenvalue extraction.

References

[1] Zienkiewicz O C and Taylor R L 1991 *The Finite Element Method*, McGraw-Hill, NY

[2] Morland H J-P and Ohayon R 1995 *Fluid Structure Interaction – Applied Numerical Methods*, John Wiley & Sons, NY

[3] Howe M S 1998 *Acoustics of Fluid-Structure Interaction*, Cambridge University Press, NY

[4] Chen S-S 1972 *Free vibration of a coupled fluid/structural system*, *J. Sound and Vibration* **21** 387

[5] Oden J T, Kikuchi N and Song Y J 1982 *Penalty finite element method for the analysis of Stokesian flows*, *Comput. Meth. Appl. Mech. Eng.* **31** 297

[6] Zienkiewicz O C and Betters P 1978 *Fluid-structure dynamic interaction and wave forces. An introduction to numerical treatment*, *Int. J. Num. Meth. Eng.* **13** 1

[7] Ding W P and Chen H L 2001 *A symmetrical finite element model for structure-acoustic coupling analysis of an elastic, thin-walled cavity*, *J. Sound and Vibration* **243** 547

[8] Everstine G C 1981 *A symmetric potential formulation for fluid-structure interaction*, *J. Sound and Vibration* **79** 157

[9] Cho J-R and Song J-M 2001 *Assessment of classical numerical models for the separate fluid-structure modal analysis*, *J. Sound and Vibration* **239** 995

[10] Sandberg G and Goransson P 1988 *A symmetric finite element formulation for acoustic fluid-structure interaction analysis*, *J. Sound and Vibration* **123** 507

[11] Cho J R and Lee H W 2004 *Numerical study on liquid sloshing in baffled tank by nonlinear element method*, *Comput. Meth. Appl. Mech. Eng.* **193** 2581

[12] Eckart C 1963 *Some transformations of the hydrodynamic equations*, *Phys. Fluids* **6** 1037

[13] Cho J R, Lee H W and Kim K W 2002 *Free vibration analysis of baffled liquid-storage tanks by the structural-acoustic finite element formulation*, *J. Sound and Vibration* **258** 847

[14] Cho J R and Oden J T 1997 *Locking and boundary layer in hierarchical models for thin elastic structures*, *Comput. Meth. Appl. Mech. Eng.* **149** 33

[15] Badur J 1984 *Numerical shell analysis via a second order approximation to the internal energy*, PhD Thesis, Institute of Fluid Flow Machinery, Polish Academy of Sciences, Gdansk

[16] Zienkiewicz O C, Too J and Taylor R L 1971 *Reduced integration technique in general analysis of plates and shells*, *Int. J. Num. Meth. Eng.* **3** 275

[17] Kekana M and Badur J 2000 *Modeling of beams using the reduced integration technique – statics and free vibration*, *Res. Dev. J.* **16** 9

[18] Badur J and Chróścielewski J 1985 *Reduced integration technique in nonlinear shell theory*, *Proc. V Conf. on "Computational Methods in Structure Mechanics", Gdynia*, pp. 12–17

[19] Chróścielewski J, Lubieniecka I and Pietraszkiewicz W 2004 *FEM and time stepping procedures in non-linear dynamics of flexible shell structures*, *Theories of Plates and Shells*, (Kienzler R *et al.* Eds) Euromech. Colloquium, **444**, pp. 21–28

[20] Oden J T and Demkowicz L F 1996 *Applied Functional Analysis*, CRC Press Inc., Boca Raton