The Influence of the Blade Length on the Natural Frequencies and Modes Shapes of Two Bladed Rigid Discs on the Shaft

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The dynamic behaviour of a rotor consisting of two bladed discs on a solid shaft is considered. The effect of shaft flexibility on the dynamic characteristics of the bladed discs and the coupling effects between the shaft and bladed disc modes are investigated. Results presented for various cases with differing blades flexibility show clearly the coupling effects in a bladed disc-shaft system.

Calculated natural frequencies obtained from blade, shaft, bladed disc and shaft with two discs are checked to discover resonance conditions and the coupling effects. The calculations show the influence of the shaft on the natural frequencies of the bladed discs up to one nodal diameter frequencies. The torsional frequency of the shaft with two discs is coupled with the zero nodal diameters modes of the single bladed discs. The bending modes of the shaft are coupled with one nodal diameter modes of the bladed discs. It is shown that including the shaft in the bladed discs model is important from the designer's point of view and can change the spectrum of frequencies considerably.

NOMENCLATURE

f	- frequency [Hz]
k	- number of nodal diameters [-]
N	- number of rotor blades [-]

INTRODUCTION

In turbomachinery the study of the vibration characteristics of individual components such as blades, discs and shafts has been established as an important part of the design. However the vibration characteristics of an individual component can change considerably when these components are assembled together to form one system, due to the coupling effects among these constituent components. The vibration characteristics of a blade in a gas or a steam turbine can change due to the flexibility of the discs and that of the shaft carrying the discs. Furthermore there may be interactions between blades on different stages of the turbine due to the flexibility of the discs and the shaft.

Two independent approaches are commonly used to analyze the dynamic behavior of turbomachinery rotating assemblies. In the first, the rotordynamics approach is concerned with disc-shaft systems. The shaft is mostly modeled by using beam finite elements and disc flexibility is not considered (Berger at el.[1]), and also the flexibility is concerned (Chivens at el.[3]). In the second approach, the bladed disc approach deals with flexible discs (Rzadkowski [4]). There are few papers of the free vibration of the bladed disc on the shaft (Dubiegeon at el.[5], Huang at el. [6], Jacquest-Richardet at el. 1996 [7], Kanki et al. [8], Khader at el. [9], Okabe at el. [10]). In these papers it is said that the inertial effects generated on the shaft by the vibration of the bladed disc are important for the modes associated with zero and one nodal diameter modes.

In this paper the natural frequencies of the single blade, the bladed disc, the discs on the shaft, the shaft and two bladed discs on the shaft are checked to discover resonance conditions and the coupling effects. The calculations show the influence of the shaft on the natural frequencies of the bladed discs up to one nodal diameter frequencies. The torsional frequency of the shaft with two discs is coupled with the zero nodal diameters modes of the single bladed discs. From the calculations it is shown that including the shaft in the bladed discs model is important from the designer's point of view and can change the spectrum of frequencies considerably. When flexible bladed discs are mounted on a flexible shaft the resultant system has vibration characteristics which depend on the coupling between the modes of vibration of the individual components. In studies of these vibration characteristics the system can not be treated as two independent systems, one being flexible bladed discs on a rigid shaft and the other being discs with rigid blades on a flexible shaft.

NUMERICAL MODEL

The structure is composed of 24 blades, mounted rigidly on a simply supported –clamped shaft with two discs (see Fig. 1). The main dimensions are as follows: the disc outer diameter is equal to 0.3808 m, the inner diameter is equal to 0.12 m, the height of the blade is 0.484 m and the length of the shaft is 3.52 m. The isoparametric brick elements with 8 nodes and 3 degrees of freedom per node are used.

Natural frequencies obtained from the cantilever blade, the shaft, the bladed disc and the shaft with two discs are checked to discover resonance conditions and the coupling effects.

In this paper geometrical cross-section blade parameters were taken from the 4th Standard Configuration [2]. The length of the blade was assumed to be L=0.484 [m] in order to get the first natural frequency close to the first natural frequency of the shaft with two discs.

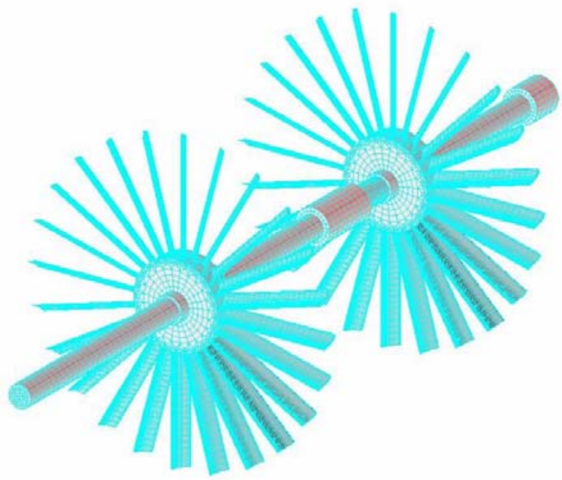

Fig. 1 Two bladed discs on the shaft

The natural frequencies of the non-rotating disc of thickness h = 0,105 [m], the inner diameters d_i = 0.12 [m]and the outer diameters d_o = 0.3808 [m] are presented in Fig. 2.

The natural frequencies of the shaft and the shaft with two discs also are presented in Fig. 2 The boundary conditions in the cylindrical coordinates (r, φ , z) from the left are z = 0, φ = 0 and in the position of bearings assumed r = 0.

The natural frequencies of the shaft with discs consist of a single torsion, the double bending and a single axial.

Thus, the natural frequencies of the non-rotating bladed disc with 24 blades of L=0.484 [m] were calculated.

The modes of the bladed disc are classified by using an analogy with axisymmetric modes which are mainly characterized by nodal lines lying along the diameters of the structure and having constant angular spacing. They are either zero (k = 0), one (k = l), two (k =2), or more (k > 2) nodal diameter bending or torsion modes. Series 1 is associated with the first natural frequency of the single centiliver blade. Series 2 is associated with the second natural frequency of the single cantilever blade; and so on k is the number of nodal diameters.

In Fig. 2, the upper axis indicates the uncoupled natural frequencies of the cantilever blade. Thus, the next axis shows natural frequencies of the bladed disc. The coupled natural frequencies of the two bladed discs on the shaft are given on the middle axis. Next, the natural frequencies of the shaft with two discs and the shaft were seen. The lower axis indicates the uncoupled natural frequencies of the disc. The numbers in brackets show the number of nodal diameters of the bladed disc.

The spectrum of natural frequencies of two bladed discs on the shaft consist of the natural frequencies connected with the natural frequencies of the first bladed disc (24 frequencies), second bladed disc (24 frequencies) and the bending and axial frequencies of the shaft with two discs. The torsion natural frequencies of the shaft with two discs are coupled with the zero nodal diameters frequencies of the bladed discs.

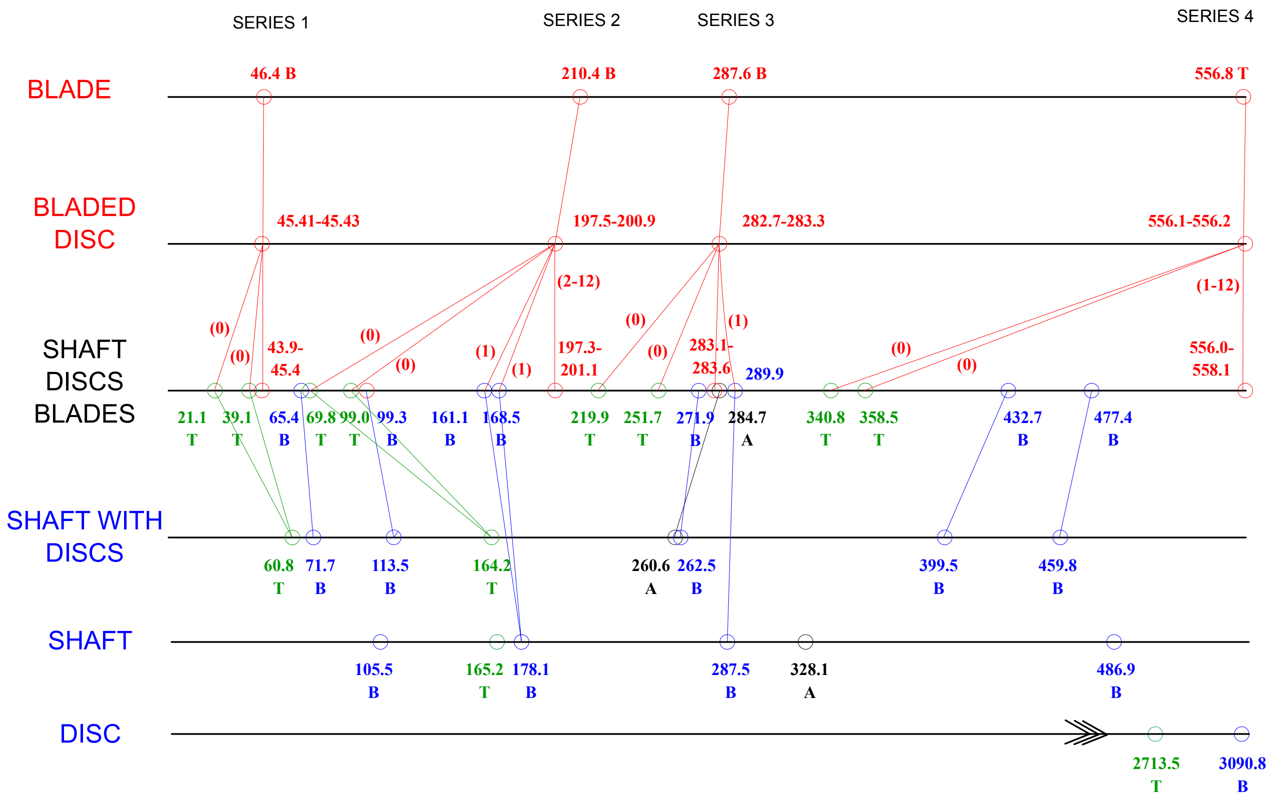

Fig.2 Natural frequencies of the non-rotating two bladed disc on the shaft (L= 0.484 m).

The uncoupled bladed disc natural frequency of the zero nodal diameter mode is 45.413 [Hz] and the uncoupled torsion natural frequency of the shaft with two discs is 60.8 [Hz]. Both are so close that the coupled natural frequency of the bladed discs on the shaft with zero nodal diameter split to values 21.1 [Hz] for the first bladed disc and 39.1 [Hz] for the second bladed disc. As shown by this example, the frequency split due to coupling effects cannot be ignored in certain situations.

The uncoupled bladed disc natural frequency of the one nodal diameters mode is 45.413 [Hz] and the coupled bending natural frequencies of the shaft with two bladed discs are 43.982 [Hz] for the first bladed disc (k=1) and 44.886 [Hz] (k=1) for the second bladed disc. It is seen that two bladed discs vibrate with one predominant. In these modes the shaft bending vibration is visible.

The uncoupled bladed disc natural frequency of the two nodal diameters mode is 45.414 [Hz] and the coupled natural frequency of the shaft with two bladed discs are in the range 45.442 [Hz] for the first and 45.442 [Hz] for the second bladed disc.

The uncoupled bladed disc natural frequencies of the three to twelve nodal diameters modes are in the range 45.414-45.434 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are in the range 45.423-45.442 [Hz] for the first and the second bladed disc. The coupled natural frequencies of the first and second bladed disc are the same for considered modes and are greater than uncoupled modes.

Next, the uncoupled shaft with two discs bending mode of the one nodal diameters is 71.7 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are 65.4 [Hz] for the first and second bladed disc (k=1). It is seen that two bladed discs vibrate with one predominant. In these modes the shaft bending vibration is visible. This coupled frequency is due to coupling effects with the bending shaft with two discs mode.

The coupled modes of the second series of the bladed discs are similar to that in the coupled first series. The torsion uncoupled mode of shaft with two discs equal to 164.2 [Hz] have an influence on the coupled natural frequencies of the bladed discs on the shaft with zero nodal diameter. These frequencies split to value 69.829 [Hz] for the first blades disc and 99.086 [Hz] for the second bladed disc. The uncoupled bladed disc natural frequency of the zero nodal diameters mode is 197.575 [Hz].

Next, the uncoupled shaft with two discs bending mode of the one nodal diameters is 113.5 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are 99.3 [Hz] for the first and second bladed discs (k=1). (Fig.2)

It is seen that two bladed discs vibrate with one predominant. In these modes the shaft bending vibration is visible. This coupled frequency is due to coupling effects with the bending shaft with two discs mode.

The uncoupled bladed disc natural frequency of the one nodal diameters mode is 197.661 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are 161.1 [Hz] for the first bladed disc (k=1) and 168.5 [Hz] (k=1) for the second bladed disc. In this case the coupling between the shaft bending mode of 178.1 [Hz] and the one nodal diameter modes of the bladed discs is observed. It is seen that two bladed discs vibrate with one predominant. In these modes the shaft bending vibration is visible.

The uncoupled bladed disc natural frequencies of the two to twelve nodal diameters modes are in the range 197.662-200.991 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are in the range 197.31-201.01 [Hz] for the first and the second bladed disc. The coupled natural frequencies of the first and second bladed disc are the same for considered modes and are less than uncoupled modes.

The coupled modes of the third series of the bladed discs are similar to that in the coupled modes of the second series.

The uncoupled bladed disc natural frequency of the zero nodal diameters mode is 282.729 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are 219.97 [Hz] for the first bladed discs (k=0) and 251.75 [Hz] (k=0) for the second bladed disc. The influence of the coupling between the blades-discs and shaft is visible here, although the uncoupled particular structure frequencies are not close in the considered region.

Next, the uncoupled shaft with two discs bending mode of the one nodal diameters is 262.5 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are 271.9 [Hz] for the first and second bladed discs (k=1). In these modes the shaft bending vibration is visible. This coupled frequency is due to coupling effects with the bending shaft with two discs.

The uncoupled bladed disc natural frequency of the one nodal diameters mode is 283.086 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are 283.6 [Hz] for the first bladed discs (k=1) and 289.9 [Hz] (k=1) for the second bladed disc. It is seen that two bladed discs vibrate with one predominant. As shown by this example, the frequency split due to coupling effects with the uncoupled shaft mode of 287.5 [Hz]. In these modes the shaft bending vibration is visible.

The uncoupled bladed disc natural frequencies of the two to twelve nodal diameters modes are in the range 283.198-283.344 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are in the range 283.18-283.39 [Hz] for the first and the second bladed disc. The coupled natural frequencies of the first and second bladed disc are the same for considered modes and are greater than uncoupled modes.

Next, the uncoupled shaft with two discs axial mode of the zero nodal diameters is 260.6 [Hz] and the coupled axial natural frequencies of the shaft with two bladed discs are 284.7 [Hz].

In order to consider the influence of the blade length on the dynamic behavior of the shaft-discs-blades system the length of the blade was decreased from L=0.484 [m] to 0.330 [m].

The geometrical cross-section blade parameters were taken from the 4th Standard Configuration [2].

In Fig.3, the left axis indicates the uncoupled natural frequencies of the cantilever blade. Thus, the next axis shows natural frequencies of the bladed disc. The coupled natural frequencies of the two bladed discs on the shaft are given on the middle axis. Next, the natural frequencies of the shaft with two discs are seen. The right axis indicates the uncoupled natural frequencies of the shaft. The numbers in brackets show the number of nodal diameters of the bladed discs modes.

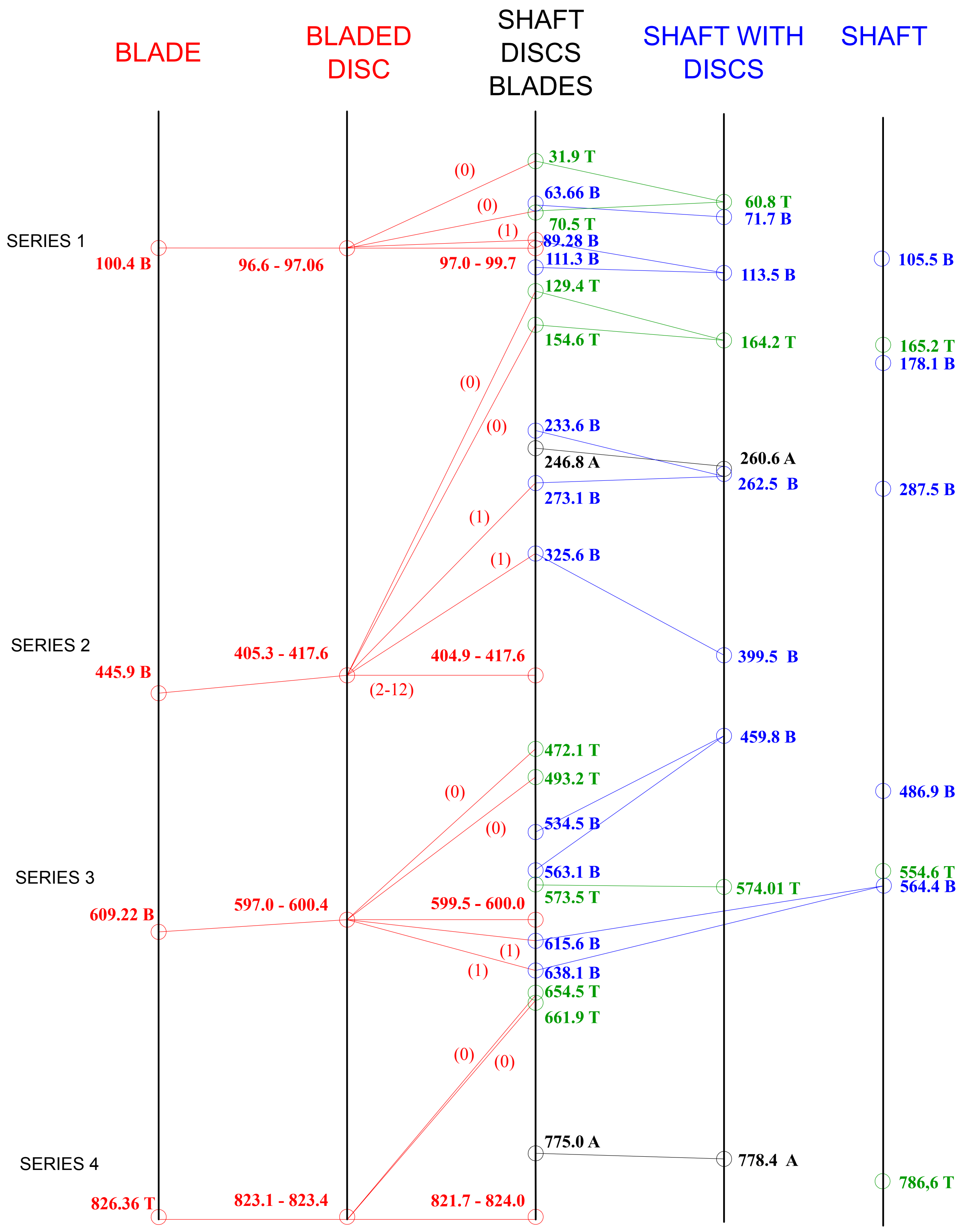

Fig. 3 Natural frequencies of the non-rotating two bladed disc on the shaft (L= 0.330 m).

The uncoupled bladed disc natural frequency of the zero nodal diameters mode is 96.969 [Hz] and the uncoupled torsion natural frequency of the shaft with two discs is 60.8 [Hz]. The coupled natural frequencies of the bladed discs on the shaft with zero nodal diameter split to values 31.998 [Hz] for the first blades disc and 70.519 [Hz] for the second bladed disc.

Next, the uncoupled shaft with two discs bending mode of the one nodal diameters is 71.7 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are 63.664 [Hz] for the first and second bladed disc (k=1).

The uncoupled shaft with two discs bending mode of the one nodal diameters is 113.5 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are 111.35 [Hz] for the first and second bladed disc (k=1).

In these modes the shaft bending vibration is visible. This coupled frequency is due to coupling effects with the bending shaft with two discs mode.

The uncoupled bladed disc natural frequencies of one to twelve nodal diameters modes are in the range 97.005-97.065 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are in the range 97.005-99.739 [Hz] for the first and the second bladed disc. The coupled natural frequencies of the first and second bladed disc are the same for considered modes and are greater than uncoupled modes. In this case the splitting of the coupled frequencies corresponding to one nodal diameter is not observed

In the next series the uncoupled bladed disc natural frequency of the zero nodal diameters mode is 405.339 [Hz] and the uncoupled torsion natural frequency of the shaft with two discs is 164.2 [Hz]. The coupled natural frequencies of the bladed discs on the shaft with zero nodal diameter split to values 129.48 [Hz] for the first blades disc and 154.66 [Hz] for the second bladed disc.

The uncoupled shaft with two discs axial mode of the zero nodal diameters is 260.6 [Hz] and the coupled axial natural frequencies of the shaft with two bladed discs are 246.8 [Hz].

The uncoupled shaft with two discs bending mode of the one nodal diameters is 262.5 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are 233.6 [Hz] for the first and second bladed disc (k=1).

The uncoupled bladed disc natural frequency of the one nodal diameters mode is 405.855 [Hz] and the uncoupled bending natural frequency of the shaft with two discs is 399.5 [Hz]. The coupled natural frequencies of the bladed discs on the shaft with one nodal diameter split to values 273.1 [Hz] for the first blades disc and 325.6 [Hz] for the second bladed disc.

The uncoupled bladed disc natural frequencies of the two to twelve nodal diameters modes are in the range 407.291-417.69 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are in the range 404.95-417.69 [Hz] for the first and the second bladed disc. The coupled natural frequencies of the first and second bladed disc are the same for considered modes and are greater than uncoupled modes.

In the third series the uncoupled bladed disc natural frequency of the zero nodal diameters mode is 597.016 [Hz]. The coupled natural frequencies of the bladed discs on the shaft with zero nodal diameters split to values 472.1 [Hz] for the first blades disc and 493.2 [Hz] for the second bladed disc.

The uncoupled shaft with two discs bending mode of the one nodal diameters is 459.8 [Hz] and the coupled bending natural frequencies of the shaft with two bladed discs are 534.5 [Hz] and 563.1 [Hz]. The uncoupled shaft frequency of 486.9 [Hz] influence these modes.

The uncoupled shaft with two discs torsion mode is 574.01 [Hz] and the coupled torsion natural frequency of the shaft with two bladed discs is 573.5 [Hz]. This shaft torsion mode does not influence the zero nodal diameters modes of bladed discs.

The uncoupled bladed disc natural frequencies of the two to twelve nodal diameters modes are in the range 599.515-600.487 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are in the range 599.2-600.49 [Hz] for the first and the second bladed disc. The coupled natural frequencies of the first and second bladed disc are the same for considered modes and are greater than uncoupled modes.

The uncoupled shaft bending mode is 564.0 [Hz] and the coupled natural frequencies of the shaft with two bladed discs are 615.6 [Hz] for the first and 638.1 [Hz] for the second bladed disc (k=1).

In order to consider the influence of the blade length on the dynamic behavior of the shaft-discs-blades system the length of the blade was decreased from L=0.330 [m] to L=0.165 [m] and L=0.094 [m]. The geometrical cross-section blade parameters were taken from the 4^{th} Standard Configuration [2]. The coupled modes of the all series of the bladed discs are similar to that in the coupled modes of the earlier cases.

In Fig.4 and Fig.5, the left axis indicates the uncoupled natural frequencies of the cantilever blade. Thus, the next axis shows natural frequencies of the bladed disc. The coupled natural frequencies of the two bladed discs on the shaft are given on the middle axis. The right indicates the uncoupled natural frequencies of the shaft with two discs. The numbers in brackets show the number of nodal diameters of the bladed discs modes. Decrease the length of blade from 0.330 [m] to 0.094 [m] causes the increase the natural frequencies of the blades – discs – shaft system. The torsion frequencies of the discs on the shaft influence the zero nodal diameter bladed disc modes considerable.

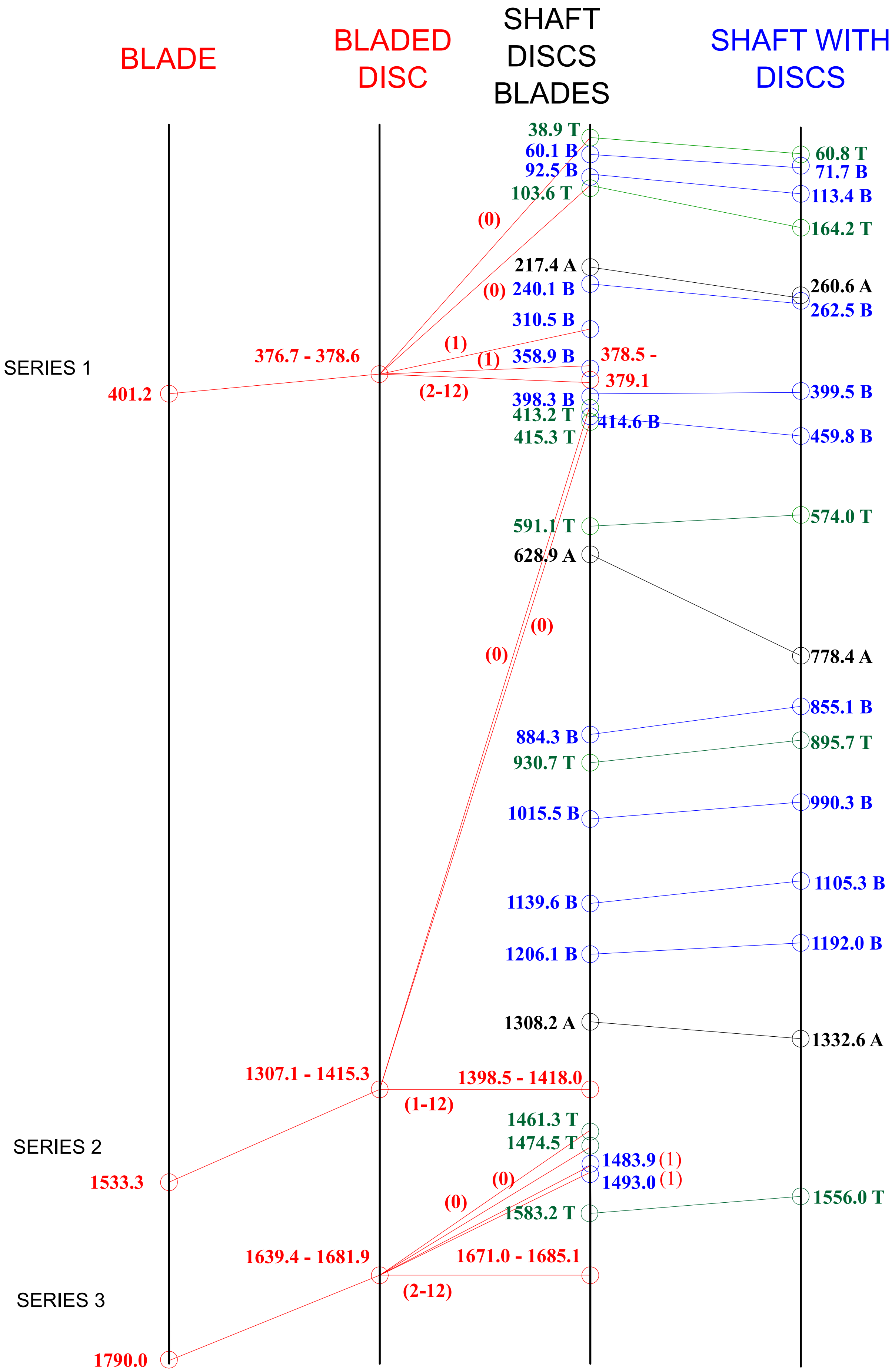

Fig. 4 Natural frequencies of the non- rotating two bladed disc on the shaft (L= 0.165 m).

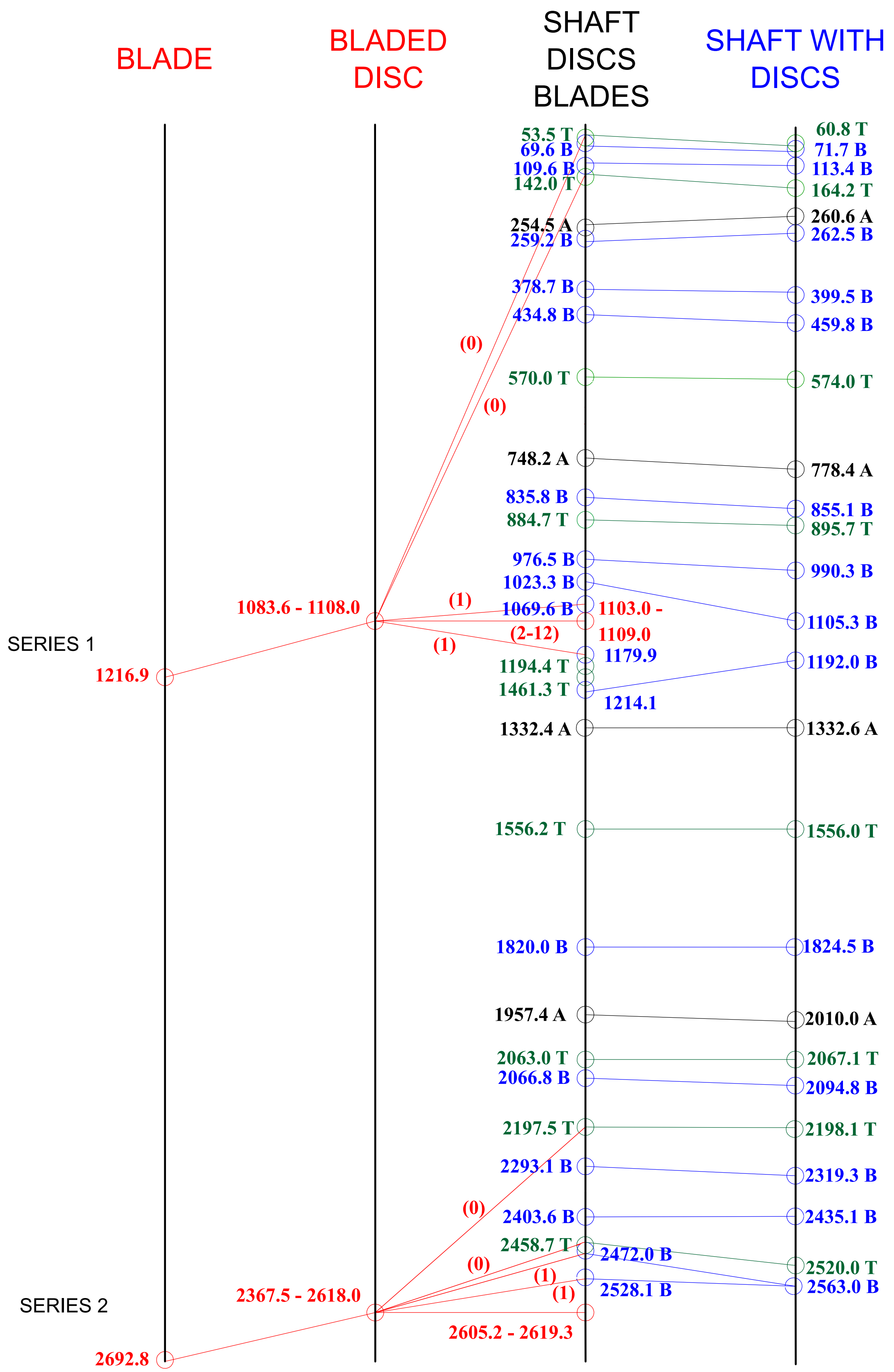

Fig. 5 Natural frequencies of the non- rotating two bladed disc on the shaft (L= 0.094 m).

CONCLUSIONS

In this paper the natural frequencies of the rotating single blade, the bladed disc, the discs on the shaft, the shaft and two bladed discs on the shaft are checked to discover resonance conditions and the coupling. The calculations show the influence of the shaft, the shaft with two discs on the natural frequencies of the bladed discs up to one nodal diameter. The torsional frequency of the shaft with two discs is coupled with the zero nodal diameters modes of the single bladed disc. If the blade dimensions are altered, the range within which strong coupling takes place may be altered. From the calculations it is shown that including the shaft in the bladed discs model is important from the designer's point of view and can change the spectrum of frequencies considerably. When flexible bladed discs are mounted on a flexible shaft the resultant system has vibration characteristics which depend on the coupling between the modes of vibration of the individual components. In studies of these vibration characteristics the system can not be treated as two independent systems, one being flexible bladed discs on a rigid shaft and the other being discs with rigid blades on a flexible shaft.

The results obtained by the 3D blades-discs-shaft model with blades of length L=0.484 [m] and L=0.330 [m] were compared with a beam-like discrete-continuous 1D mechanical model [11] for bladed disc modes with zero nodal diameters (Tab. 1). The bladed discs were be substituted in the 1-D discrete-continuous model by the system of dynamic oscillators in the form of rigid rings mutually attached to the rotor-shaft by means of the visco-elastic mass-less membranes enabling rotations of these rings as well as their translational displacements in the shaft axial direction. Parameters of these oscillators have been determined by the use of the proper reduction method described in [11]. The boundary conditions in the cylindrical coordinates (r, φ, z) from the left are z = 0 and in the position of bearings assumed r = 0. The comparison of the results of the 3D and 1D models are not satisfactory, although it is seen that the zero nodal diameters modes are relatively close.

Tab.1 The natural frequencies of the bladed discs-shaft of 3D and 1D models for modes of zero nodal diameters.

	L = 0.484 [m]		L = 0.330 [m]	
Lp.	[11] [Hz]	NASTRAN 3D [Hz]	[11] [Hz]	NASTRAN 3D [Hz]
1	41.937	37.377	74.978	64.475
2	59.879	65.192	118.980	126.590
3	92.904	91.422	136.969	146.840
4	223.270	219.520	461.921	469.620
5	226.118	250.200	466.899	492.410

Tab.2 The natural frequencies of the bladed discs-shaft of 3D and 1D models for modes of one nodal diameters.

	L = 0.484 [m]		L = 0.330 [m]	
Lp.	[11] [Hz]	NASTRAN 3D [Hz]	[11] [Hz]	NASTRAN 3D [Hz]
1	46.128	43.837	98.890	99.566
2	46.237	43.837	98.989	106.45
3	210.011	167.08	443.139	404.96
4	210.327	167.08	443.562	404.96

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