REMOVAL OF SUBMICROMETER DUST PARTICLES 
BY A CHARGED SPHERICAL COLLECTOR 

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Abstract
A numerical model for determining 3D trajectories of charged dust particles in the vicinity of a falling down, oppositely charged spherical collector (a droplet) is presented. A new definition of the collection efficiency is also proposed. The model is applied to determine the limiting collision trajectories for dust particles, the precipitation space and collection efficiency for a single charged spherical collector.

1. Introduction
The difficulty experienced in the removal of submicrometer dust particles by conventional inertial scrubbers lies in low inertial forces on dust particles smaller than a few \(\mu\)m in diameter. Charging the dust particles and the scrubbing droplets to opposite polarities removes this shortcoming and allows removal dust particles of micrometer and submicrometer size range from exhaust gases. The charged droplets act as small spherical collecting electrodes uniformly distributed in the scrubber. The distances of the dust particles to these collectors are very short, and the attractive Coulomb force easily causes the particles to move towards a charged droplet up to mechanical contact with it. The scrubbing processes utilizing electrostatic forces may operate at lower relative velocities between dust particles and the droplet than those in which inertial collection is dominant.

In the most recent theoretical studies of the electrostatically enhanced scrubbing processes, the computational domain of the particle - collector interactions was usually reduced to two dimensions and the collector was fixed. A more realistic situation is to assume that the drop is falling down under the gravity and taking up the velocity of the surrounding gas. Conditions in the case of the moving collector are quite different from those of the fixed one: the flow field around the drop changes with time, and the inertial forces may dominate the electrostatic attraction.

The purpose of this paper is to report the results of numerical modeling of particle trajectories in the vicinity of a charged spherical collector, and to calculate the precipitation space and collection efficiency for a single collector which traverses a flowing cloud of opposite charged dust. In the present model three dimensional equations of motion are considered, including the motion of the collector. Simulation of the dust particle flow in the vicinity of the moving collector, in the 3D space can provide much more information on the complex nature of the scrubbing process. A new definition of the collection efficiency, which considerably differs from that commonly used in the literature definition of Kraemer and Johnstone [1], is proposed in this paper.
2. Equations of motion

The dust particle trajectories in the vicinity of a falling down charged spherical collector (a drop), and the precipitation space for this collector are determined numerically by simultaneous solving the equations of motion of the particle and the collector. It is assumed that the Coulomb, Stokes, gravitational and inertial forces act upon the dust particle. The image and polarization forces as well as thermo- and diffusiophoresis are not considered. It is also assumed that both the droplet and dust particle are spherical, and the flow field in the vicinity of the collector is not disturbed by the dust particles. The origin of the coordinate set is fixed at the centre of the collector.

The trajectory of a dust particle of mass \( m \) in the vicinity of the spherical collector charged with the charge \( Q_c \) is given by the following vector differential equation:

\[
\frac{d\mathbf{w}}{dt} = \frac{C_d}{24} \frac{\Re_p}{R_p} \mathbf{F}_S + \mathbf{F}_e + \mathbf{F}_g
\]

where \( \mathbf{w} \) is the particle velocity, \( \mathbf{F}_g \) - the gravity force, \( \mathbf{F}_S \) - the Stokes drag force:

\[
\mathbf{F}_S = 6\pi\eta_g R_p (\mathbf{u} - \mathbf{w}) / C_c
\]

and \( \mathbf{F}_e \) is the electrostatic force on the particle.

\[
\mathbf{F}_e = \frac{Q_p Q_c}{4\pi\varepsilon_0 R_c^2}
\]

For \( \Re_p < 2 \times 10^5 \) the non-Stokesian drag coefficient \( C_d \) is given by the Brauer and Sucker [2] equation:

\[
C_d = 24 \Re_p^{-1} + 3.73 \Re_p^{-1/2} - \frac{4.83 \times 10^{-3} \Re_p^{1/2}}{1 + 3 \times 10^{-6} \Re_p^{3/2}} + 0.49
\]

The geometrical dimensions can be normalized to the collector radius \( R_c \). All velocities, i.e., gas velocity \( \mathbf{u} \), dust particle velocity \( \mathbf{w} \), and collector velocity \( \mathbf{v} \) can be normalized to the gas velocity \( \mathbf{u}_0 \) in the undisturbed region. Time can be converted into dimensionless form by \( \tilde{t} = tu_0 / R_c \).

The Coulomb force normalized to Stokes drag force on the dust particle is known as the Coulomb number:

\[
K_c = \frac{C_c Q_p Q_c}{24\pi^2 \eta_g u_0 \varepsilon_0 R_c^2 R_p}
\]

The Stokes number is given by the equation:

\[
St = \frac{2C_c R_p^2 \rho_p u_0}{9\eta_g R_c}
\]

where \( \rho_p \) is the particle density.
In dimensionless form the equation of motion of the particle becomes:

$$\frac{d^2 r}{d\tau^2} = \frac{1}{St} \frac{C_d}{24} \left( u - \frac{dr}{d\tau} \right) + \frac{Kc}{St} \frac{r}{|r|} + g \frac{R}{u_0^2}$$ (7)

The flow field around the spherical collector is simulated by means of the stream function $\Psi$ given by Hadamard and Rybczyński [3]:

$$\Psi = -\frac{v_c}{2} \left( 1 - \frac{1}{2} \frac{2 + 3\kappa}{1 + \kappa} + \frac{\kappa}{2} \frac{1}{(1 + \kappa)^2} \right) \sin^2 \Theta,$$ (8)

$$\kappa = \mu_p / \mu_g$$

Both components of the velocity vector can be calculated using the Oseen system of two differential equations, which in spherical coordinates $r, \Theta$ are:

$$u_r = \frac{d\Psi}{d\tau} = \frac{1}{r^2 \sin \Theta} \frac{\partial \Psi}{\partial \Theta}$$ (9)

$$\frac{d\Theta}{d\tau} = -\frac{1}{r \sin \Theta} \frac{\partial \Psi}{\partial r}$$ (10)

$$v_c = \left( \frac{1 + \nu_x}{\nu_c} \right)^{\nu_x/2}$$ (11)

The collecting drop enters the scrubbing channel with the initial velocity $v_y = v_{yo}$ and $v_x = 0$, and is accelerated due to the gravity to the velocity $v_y$, and by the flowing gas to velocity $v_x$. The drop velocity is given by a system of differential equations [4]:

$$\frac{d\nu_x}{d\tau} = 1 - \frac{3}{8} \frac{\rho_g}{\rho_c} \nu_c c_x$$ (12)

$$\frac{d\nu_y}{d\tau} = g \frac{R}{u_0^2} - \frac{3}{8} \frac{\rho_g}{\rho_c} \nu_c v_c c_x$$ (13)

where $c_x$ is the drag coefficient, for Reynolds numbers based on the collector. For $Re_c \leq 10^5$, $c_x$ is approximated by the Kaskas equation [4]:

$$c_x = 24 Re_c^{-1} + 4 Re_c^{-1/2} + 0.4$$ (14)

3. Computational and experimental results

Equation (7), and simultaneously equations (12) and (13), have been solved numerically with the fourth order Runge-Kutta method and following initial conditions $(t=0) x=x_0, y=y_0, z=z_0, w_x=u_0, w_y=0, w_z=0$ for the dust particle, and $v_x=0, v_y=v_{yo}, v_z=0$ for the collector. For the initial values of the Coulomb number $Kc=10$ and $Kc=0$, Stokes number $St=0.1, St=1$ and $St=10$, and initial conditions $x(t=0)=-20R_c$ and $z(t=0)=0$ the trajectories of the dust particles approaching the spherical collector are presented in Fig.1. Only those particles whose trajectories terminate at the collector surface can be captured, and then removed from the gas stream.

The precipitation space is defined as the geometrical loci of those starting points of dust particle trajectories which terminate at the surface of the collector. Fig.2 shows the numerically determined precipitation space for different initial values of the Stokes and Coulomb numbers. The precipitation space also includes the collector itself, and is closed downstream the collector.
Fig. 1. Dust particle trajectories in the vicinity of the charged spherical collector in the plane $z=0$ ($R_c=0.5\text{mm}$, $u_o=0.5\ \text{m/s}$, $v_{yo}=0.5\ \text{m/s}$).

Fig. 2. Precipitation space for a single charged spherical collector ($R_c=0.5\text{mm}$, $u_o=0.5\ \text{m/s}$, $v_{yo}=0.5\ \text{m/s}$).
The collection efficiency is defined in this paper as the relation of the volume $V_p$ of the precipitation space to the volume $V_s$ swept by the collector relative to flowing gas:

$$Ef = \frac{V_p}{V_s} \tag{15}$$

Both $V_p$ and $V_s$ are determined numerically from the solution of equation (7). The numerical results of collection efficiency together with the results of measurements are presented in Fig.3. The measurements were carried out at an experimental stand presented in papers [5,6].

![Fig.3. Surface charge density effect on the collection efficiency](image)

![Fig.4. Collector radius effect on the collection efficiency](image)

The definition (15) differs from that of Kraemer and Johnstone [1]. The collection efficiency defined by Kraemer and Johnstone was determined as the ratio of the cross-section for the limiting trajectories starting at infinity, to the cross-section of the collector. In practice it is impossible to ensure if the starting point of the trajectory is sufficiently distant from the collector. Our definition is more suitable for real
situations because the assumptions that the collector is fixed, and the relative velocity is constant are not necessary. It is also insensitive to the initial conditions such as the starting point of the dust particle, because all potential collisions are considered.

4. Conclusions

The precipitation space and collection efficiency for a single charged droplet falling down in a flowing cloud of oppositely charged dust particles have been determined numerically. Charging both the dust particles and the collecting droplet causes an increase in the volume of the precipitation space and the collection efficiency.

The precipitation space and the collection efficiency are affected by the Stokes number which is mainly the function of the gas - collector relative velocity, and Coulomb number which depends principally on the charges on the collector and dust particle.

The deposition of dust particles on a moving spherical collector due to the electrostatic forces are dominant only for small Stokes numbers. With an increase in Stokes number the electrostatic effect diminishes. The Coulomb number and collection efficiency increase for small collecting droplets charged close to the Rayleigh limit, for example when an electrohydrodynamic method is used.

It should be mentioned that in the inertial scrubbers the relative velocities between the collector and dust particle should be as high as possible to overcome the viscosity effect. However, because the collector takes up the velocity of the surrounding gas, specifically for smaller droplets, this is not easy to achieve. In the charged droplet scrubbers this conflict does not exist. With a decrease in the relative velocity the volume of the precipitation space, and then the collection efficiency increases.

A new definition of the collection efficiency, different from those used in the literature, and new method for its determination have been proposed.

References