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QUANTUM ELECTRONICS

Influence of Parabolical Radial Gain Distribution of a Medium with Gain Saturation on Laser Modes. I

by

J. MIZERACZYK and J. CHOJNACKI

Presented by R. SZEWALSKI on July 8, 1974

Summary. Approximate theory of a laser resonator filled with a non-linear active medium is presented under the assumption that the gain distribution over the transverse cross-section of the resonator is parabolic. The paper includes calculations for media having uniform transverse gain. The results suggest that the mode patterns are identical in both cases. It means that the influence of a parabolic radial gain distribution on the field distribution in the active confocal resonator is negligible within the gain range examined.

1. Introduction. Electromagnetic field distribution over mirrors of an active laser resonator and resonant frequency of such a resonator depend on its geometry and on parameters of an amplifying medium the resonator is filled with.

It is due to difficulties, mainly mathematical, resulting from taking into consideration the effect of propagation of radiation in a non-linear and non-uniform amplifying medium, that the problem of determination of field distribution and resonant frequencies of a laser resonator — the parameters of which determine a mode (type of oscillations) of the resonator — is usually confined to solving the problem for a passive resonator. This means that only the resonator geometry influence on the modes is taken into account whereas the influence of the amplifying medium is neglected [1]. Such simplification of the active laser resonator theory does not fully describe generating features of the laser resonator-active medium system.

The paper by Li and Skinner [2] presents one of the earliest approaches to the problem of a real active laser resonator. Nonuniformity of a linear amplifying medium transverse to the resonator axis was taken into account there. It was shown that in the case of small nonuniformities of the amplifying medium the modes of an active resonator under consideration differ but slightly from those of the corresponding passive resonator.

The further step into the theory of the active resonator was taken by Statz and Tang [3] who assumed non-linear features of the amplifying medium, resulting from the gain saturation due to the increase of radiation intensity within the resonator. Owing to mathematical difficulties involved an assumption was made that transversely uniform amplifying medium was concentrated in two infinitely thin sheets next to the surfaces of parallel-plane mirrors. In this model of the active resonator the radiation is propagating between the mirrors through a passive medium, whereas gain takes place directly on the mirrors. The above formulation of the problem of propagation and gain in a laser resonator simplified mathematics of the problem. The results obtained show that the modes of the resonator are negligibly affected by gain effects.

Fox and Li [4] extended the approximation of Statz and Tang onto the case of a parallel-plane and confocal resonators immersed in a transversely uniform non-linear amplifying medium. Their results of numerical calculations for a medium with small gain proved those basic properties of a mode, e.g. field distributions over the resonator mirrors, diffraction losses and resonant frequencies of the resonator are identical with those obtained for corresponding passive resonator. Locchi [5] obtained similar results for a parallel-plane resonator filled with transversely uniform non-linear amplifying medium.

Presented below is an approximate theory of a laser resonator filled with an active medium characterized by the parabolical gain distribution, with nonlinearity resulting from gain saturation taken into consideration. Parabolical gain distribution was chosen as an approximation of actual gain distributions in some laser media [6]. Also given are the results of numerical calculations for a confocal resonator filled with such an amplifying medium.

The aim of this paper was to obtain the information regarding simultaneous influence of a non-uniform gain distribution transverse to the resonator axis and a gain non-linearity caused by gain saturation due to increase of radiation intensity inside the resonator.

2. Mathematical formulation. The analysis will deal with a symmetrical laser resonator consisting of two identical spherical (or flat) mirrors Z_1 and Z_2 (Figure). The distance between them equals d. Radii of curvature are identical and equal to R. The mirrors are of circular shape with $2a$ diameter. It is assumed that all the resonator dimensions are large as compared to the wavelength of laser radiation and that the inequality

$$a \ll d, \tag{1}$$

typical of laser resonators is valid. The whole space between the mirrors is filled with a medium, the gain coefficient of which $\alpha(r, w)$ is given by the formula [7]

$$\alpha(r, w) = \frac{\alpha_0(r)}{\left(1+\frac{w}{w_0}\right)^{\nu}}, \tag{2}$$

where $\alpha_0(r)$ is the unsaturated gain coefficient (for $w \ll w_0$), w is the radiation intensity and w_0 is a saturation parameter. The exponent ν is equal to $\frac{1}{2}$ or 1 for the inhomogeneously or homogeneously broadened line, respectively.

The gain coefficient of the medium $\alpha(r, w)$ is defined by the formula

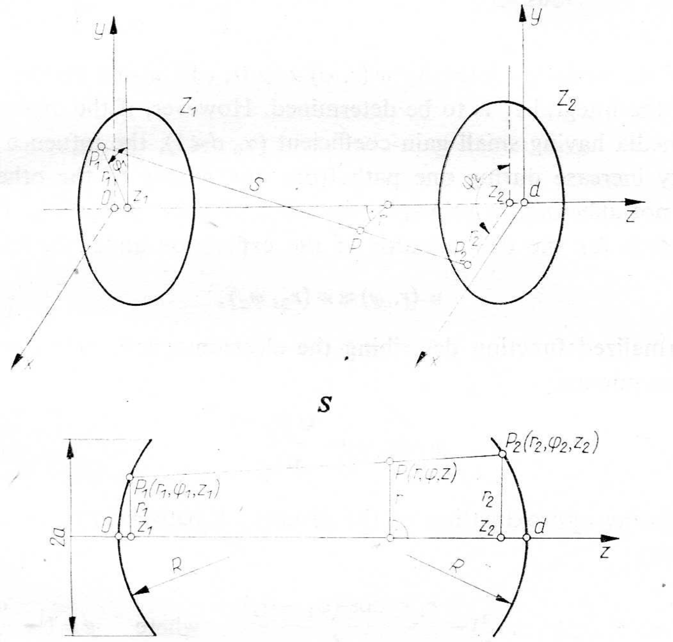
$$dw = \alpha w dz, \tag{3}$$

where dw is the intensity increment after passing the medium along dz distance in the direction of propagation.

It is assumed that the gain coefficient of the medium varies parabolically in the radial direction, i.e. that the $\alpha_0(r)$ coefficient is expressed as follows:

$$\alpha_0(r) = \alpha_0 \left(1 - \xi \frac{r^2}{a^2}\right), \tag{4}$$

where α_0 is the unsaturated gain coefficient along the resonator axis (for $w \ll w_0$), whereas ξ is a characteristic parameter of the medium. The above assumption concerning the gain distribution is justified by the fact that this distribution describes adequately the actual gain in many laser media [6]. The $\alpha_0(r)$ coefficient is frequency dependent [7, 8]. This, however, was not taken into account in our analysis.



Geometry of the symmetrical laser resonator

The Huygens—Fresnel principle [9] allows expressing the electromagnetic field distribution $\psi(r_1, \varphi_1)$ over the mirror Z_1 of the resonator considered as follows:

$$\psi(r_1, \varphi_1) = \frac{ik}{2\pi} \rho \int_0^a \int_0^{2\pi} \psi(r_2, \varphi_2) \frac{\exp\left(-i \int_{P_2(r_2, \varphi_2)}^{P_1(r_1, \varphi_1)} \tilde{k}[P(r, \varphi)]\, ds\right)}{S} r_2\, dr_2\, d\varphi_2, \tag{5}$$

where

$$\tilde{k}(r, \varphi) = k + i \frac{1}{2} \alpha [P(r, \varphi)] \tag{6}$$

is the complex propagation constant k — the wave number and ρ^2 is the mirror reflectivity. The function $\psi(r_2, \varphi_2)$ describes the field distribution over the mirror Z_2. Remaining quantities that appear in the integral equation are defined in the Figure.

The integral present in the exponent of the expression under the integral (5) can be expressed as follows:

(7) $$I = \int_{P_2(r_2,\varphi_2)}^{P_1(r_1,\varphi_1)} \tilde{k}[P(r,\varphi)]\, ds = k\cdot S + i\frac{1}{2} I_1,$$

where

(8) $$I_1 = \int_{P_2(r_2,\varphi_2)}^{P_1(r_1,\varphi_1)} \alpha(r, w)\, ds = \alpha_0 \int_{P_2(r_2,\varphi_2)}^{P_1(r_1,\Phi_1)} \frac{1-\xi\dfrac{r^2}{a^2}}{\left[1+\dfrac{w(r,\varphi)}{w_0}\right]^{\nu}} ds.$$

The value of the radiation intensity $w(r, \varphi) \propto |\psi(r, \varphi)|^2$ inside the resonator should be known if the integral I_1 is to be determined. However, if the analysis will be restricted to media having small gain coefficient ($\alpha_0 d \ll 1$), the infuence of the radiation intensity increase during one path from one mirror to the other can be recognized as not affecting, in principle, the effect of gain saturation. It will be assumed therefore for the denominator of the expression under the integral (8) that

(9) $$w(r, \varphi) \approx w(r_2, \varphi_2).$$

Using a normalized function describing the electromagnetic field distribution over the resonator mirrors:

(10) $$\psi^*(r, \varphi) = \frac{\psi(r, \varphi)}{\sqrt{w_0}},$$

and the following approximation of the distance S between two points $P_1(r_1\ \varphi_1)$ and $P_2(r_2, \varphi_2)$

(11) $$S \approx d + \frac{g}{2d}(r_1^2 + r_2^2) - \frac{r_1 r_2 \cos(\varphi_1 - \varphi_2)}{d} \quad \text{where} \quad g = 1 - \frac{d}{R},$$

one obtains [10]

(12) $$I_1 = \frac{\alpha_0}{[1+|\psi^*(r_2, \varphi_2)|^2]^{\nu}} \left[d + \frac{g-2\beta}{2d}(r_1^2 + r_2^2) - \frac{r_1 r_2 (1+\beta)}{d} \cos(\varphi_1 - \varphi_2)\right].$$

Here

(13) $$\beta = \frac{\xi d^2}{3a}.$$

Taking approximately $S \cong d$ in the denominator of the expressi on under the integral in Eq. (5) one obtains:

(14) $$\psi(r_1, \varphi_1) = \rho \int_0^a \int_0^{2\pi} \mathscr{K}[r_1, \varphi_1, r_2, \varphi_2, \psi^*(r_2, \varphi_2)]\, r_2\, dr_2\, d\varphi_2.$$

where the kernel $\mathscr{K}$ of the integral equation has the form:

(15) $$\mathscr{K}[r_1, \varphi_1, r_2, \varphi_2, \psi^*(r_2, \varphi_2)] = \mathscr{K}^{(1)}(r_1, \varphi_1, r_2, \varphi_2)\, \mathscr{K}^{(2)}[r_1, \varphi_1, r_2, \varphi_2, \psi^*(r_2, \varphi_2)].$$

The function

(16) $$\mathscr{K}^{(1)}(r_1, \varphi_1, r_2, \varphi_2) = \frac{ik}{2\pi d} \exp\left\{-\frac{ik}{2 \cdot d}[g(r_1^2 - r_2^2) - 2r_1 r_2 \cos(\varphi_1 - \varphi_2)]\right\}$$

is the kernel of the symmetrical passive resonator [11], whereas the function

(17) $$\mathscr{K}^{(2)}[r_1, \varphi_1, r_2, \varphi_2, \psi^*(r_2, \varphi_2)] = \exp\left(\frac{1}{2} I_1\right)$$

describes the gain the wave is subjected to when passing from the point $P_2(r_2, \varphi_2)$ to the point $P_1(r_1, \varphi_1)$. It can be assumed for a medium with small gain ($\alpha_0 d \ll 1$) that

(18) $$\mathscr{K}^{(2)} = \exp\left(\frac{1}{2} I_1\right) \approx 1 + \frac{1}{2} I_1 .$$

For axisymmetric solutions of Eq. (15), i.e. for the solutions where

$$\psi^*(r, \varphi) = R_n(r) \exp[-in\varphi], \quad n \text{ — integer},$$

the expression (18) can be averaged over an angle $\varphi = \varphi_1 - \varphi_2$. The function thus obtained describes the average gain along a path from any point of the circle the point $P_2(r_2, \varphi_2)$ belongs to, to any point of the circle that comprises the point $P_1(r_1, \varphi_1)$. The average gain $\overline{\mathscr{K}}^{(2)}$ is equal to

(19) $$\overline{\mathscr{K}}^{(2)}[r_1, r_2, R_n(r_2)] = 1 + \frac{\alpha_0\left[d + \frac{g - 2\beta}{2d}(r_1^2 + r_2^2)\right]}{\{2\ 1 + [R_n(r_2)]\}^{\nu}} .$$

Further, using the relation [12]:

(20) $$\exp\left[in\left(\frac{\pi}{2} - \beta\right)\right] \cdot In(x\, y) = \frac{1}{2\pi} \int_0^{2\pi} \exp\{i[x\, y \cos(\alpha - \beta) - n\alpha]\}\, d\alpha$$

one obtains the following integral equation for the function $R_n(r)$

(21) $$R_n(\eta_1) = \rho \int_0^1 \mathscr{K}_n^{(1)}(\eta_1, \eta_2) \cdot \overline{\mathscr{K}}^{(2)}[\eta_1, \eta_2, R_n(\eta_2)]\, R_n(\eta_2)\, d\eta_2, \quad \text{where} \quad \eta = \frac{r}{a},$$

(22) $$\mathscr{K}_n^{(2)} = i^{n+1}\, 2\pi N \eta_2\, I_n(2\pi N \eta_1\, \eta_2) \exp[-iN\pi g(\eta_1^2 + \eta_2^2)],$$

(23) $$\overline{\mathscr{K}}^{(2)} = 1 + \frac{\alpha_0\, d\left[1 - \frac{\xi}{3}(\eta_1^2 + \eta_2^2)\right]}{2\{1 + [R_n(\eta_2)]^2\}^{\nu}} \qquad N = \frac{a^2}{\lambda d} \quad \text{(Fresnel number)},$$

whereas I_n is the Bessel function of the first kind and n-th order.

Eq. (21) is a non-linear integral equation. It can be solved numerically by an iterative method*) in a manner similar as used by Fox and Li [4, 13] for a passive

*) These numerical calculations have been carried out, and discussed in detail, in Part II of this paper (this issue).

resonator as well as for an active one filled with a medium characterized by a uniform gain distribution. Eq. (21) allows also calculation of field distribution for a cavity with uniform gain distribution, because for $\xi=0$ it becomes identical to the equation of Fox and Li [4] that concerns the field distribution in a resonator filled with a non-linear medium having a uniform gain distribution.

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REFERENCES

[1] H. Kogelnik, T. Li, *Laser beams and resonators*, Proc. IEEE, **54** (1966), 1312—1329.

[2] T. Li, J. G. Skinner, *Oscillating modes in ruby lasers with nonuniform pumping energy distribution*, J. Appl. Phys., **36** (1965), 2595—2596.

[3] H. Statz, C. L. Tang, *Problem of mode deformation in optical masers*, ibid., 1816—1819.

[4] A. G. Fox, T. Li, *Effect of gain saturation on the oscillating modes of optical masers*, IEEE., J. Quantum Electronics, **QE-2**, (1966), 774—783.

[5] F. Locchi, *Oscillating modes in open resonators with saturable gain medium*, Atti. Fond. Giorgio Ronchi Contrib. Ist. Naz. Ottica, **26** (1971), 641—659.

[6] W. R. Bennett, *Excitation mechanism in gas lasers*, Appl. Optics, Suppl., Chemical Lasers, **3** (1965); See also: *Gazovye lazery*, Sbornik Statiej, Mir, Moskva, 1968.

[7] W. W. Rigrod, *Gain saturation and output power of optical masers*, J. Appl. Phys., **34** (1963), 2602—2609.

[8] P. W. Smith, *The output power of a* 6328 A *gas laser*, IEEE, J. Quantum Electronics, **QE-2** (1966), 62—68.

[9] M. Born, W. Wolf, *Principles of optics*, Pergamon Press, London—Los Angeles, 1959.

[10] J. Mizeraczyk, *Equivalence relations for a laser resonator with a lens-like medium — approximation resulting from the Huygens-Fresnel principle*, Acta Phys. Pol., **42A** (1972), 147—154.

[11] T. Li, *Diffraction loss and selection of modes in maser resonators with circular mirrors*, BSTJ, **44** (1965), 917—932.

[12] J. A. Stratton, *Electromagnetic theory*, McGraw Hill., Inc. New York and London, 1941.

[13] A. G. Fox, T. Li, *Resonant modes in a maser interferometer*, BSTJ, **40** (1961), 453—488.

[14] J. Chojnacki, *Numeryczna analiza nieliniowach równań całkowych w teorii rezonatorów optycznych*, Biul. IMP PAN (1974), [in press].

[15] J. Mizeraczyk, *Metody wyznaczania rozkładów pola częstotliwości drgań własnych rezonatorów laserowych*, ibid., **15** (1971), 684.

[16] — , *Wpływ konfiguracji rezonatora optycznego na strukturę przestrzenną wiązki promieniowania*, ibid., **26** (1971), 688.

Е. Мизерачик, Е. Хойнацки, **Влияние параболического радиального распределения среды с насыщением на моды лазера. I**

Содержание. Представлена приближенная теория лазерного резонатора с нелинейной активной средой характеризующейся параболическим распределением коэффициента усиления в поперечном сечении. Работа содержит также расчеты для среды с однородным распределением усиления. Получено одинаковое распределение поля на зеркалах для обоих рассмотренных случаев. Следует предполагать, что влияние параболического радиального распределения усиления на распределение поля в активном резонаторе в исследованном диапазоне усиления ($\alpha_0 d < 15\%$) пренебрежимо мало.

Note. The list of the References deals also with Part II of this paper.

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12.

QUANTUM ELECTRONICS

Influence of Parabolical Radial Gain Distribution of a Medium with Gain Saturation on Laser Modes. II

by

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Presented by R. SZEWALSKI on July 8, 1974

Summary. The paper comprises the results of the numerical calculations of the field distributions over the mirrors of a confocal resonator, filled with such an active medium, made on the basis of an integral equation being derived in Part I of this paper (cf. this issue, pp. 113 [1073]). The paper includes also the final remarks which refer to both parts of this work.

1. Numerical results for the confocal resonator. The results presented in the paper were obtained with the use of an Odra 1204 computer. The iterative method [14] was employed. It was due to long duration of calculations that Eq. (21) was solved, in principle, only for a simpler case ($N=1$) of a confocal resonator ($g_1=g_2=0$) widely referred to in the bibliography. The distance between the confocal resonator mirrors was assumed to be 1 m. Only a medium with homogeneously broadened line ($\nu=1$) was taken into consideration.

The calculations were made parallelly for two cases: $\xi=0$ (uniform transverse gain distribution) and $\xi=1$ (parabolical transverse gain distribution), in order to determine the influence of non-uniform gain distribution on mode parameters.

Fig. 1 shows a relative field distribution over the mirrors of a confocal resonator ($N=1$) for the two cases of gain distribution $\xi=0$ and $\xi=1$. The calculations were made for selected values of unsaturated gain per path $\alpha_0 d$ from the range (0—15%) and various transmittances $T (T=1-\rho^2)$ of a mirror; the value of T was always chosen lower than that of $\alpha_0 d$. The range of $\alpha_0 d$ was chosen to fulfil the condition $\alpha_0 d \ll 1$, assumed when deriving the integral equation (21). Numerical results, the diagram in Fig. 1 was based upon, are listed in the Table. In turn, Fig. 2 shows a relative field distribution over the confocal resonator mirrors ($N=1.5$) for ξ equal to 0 and 1, $T=1\%$ and $\alpha_0 d \doteq 10\%$.

It is evident from the comparison of the results presented in the Table and in Figs. 1 and 2 that the relative field distributions over the mirrors of a confocal resonator ($N=1$ and 1.5) are in both cases, $\xi=0$ and $\xi=1$, identical, up to the third

Note. The list of References, given in Part I concerns both the parts of this paper.

significant figure. This means that within the range of $\alpha_0\, d$ variability taken into account the influence of gain amplitude and distribution on the relative field distributions is of secondary importance as compared to the deciding influence of the Fresnel number N. The influence of the Fresnel number N on the relative field distribution was illustrated in Fig. 1, where the relative field distribution over the mirrors of a passive confocal resonator ($\alpha_0\, d=0$, $T=0\%$) was shown as determined for

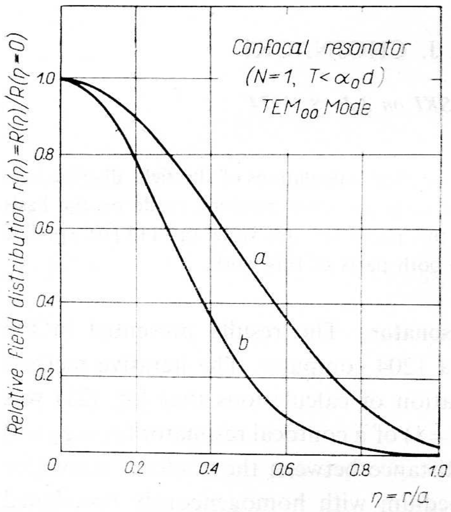

Fig. 1. Relative field distribution over the mirrors of a confocal resonator for $\xi=0$ and $\xi=1$ — TEM_{00} mode

a) $\alpha_0\, d=0$; 2.5—15%, $\xi=0.1$; *b*) gaussian distribution ($N\to\infty$)

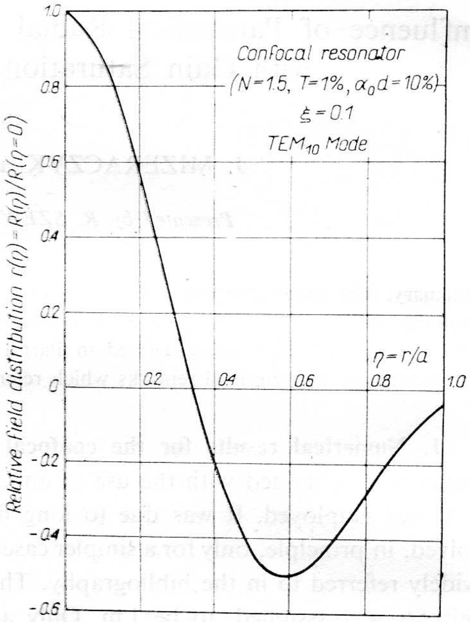

Fig. 2. Relative field distribution over the mirrors of a confocal resonator for $\xi=0$ and $\xi=1$ — TEM_{10} mode

the limit case of infinite transverse dimensions of the mirrors, i.e. for $N\to\infty$. This distribution is of Gaussian type (see [1, 15, 16]). Further, it can be observed that the relative field distribution, up to the third significant figure, is not affected by mirror transmittances.

The results obtained for $\xi=0$, $\alpha_0\, d=10\%$, $T=1\%$, $N=1$ or 1.5 were compared with those of Fox and Li [4] obtained for the same set of parameters. Both groups of results are indistinguishable within the limits of diagram accuracy.

The calculations presented in this paper have shown that in both cases: $\xi=0$ and $\xi=1$, for $\alpha_0\, d$ within the limits (0, 15%), the resonator mode that predominates for $N<1.3$ is the TEM_{00} mode, whereas for $N>1.3$ predominating is the TEM_{10} mode. These results confirm the results obtained earlier by Fox and Li [4], namely, that for a resonator with uniform transverse gain distribution of a medium $N=1.32$ is a point of quasi-equilibrium between the two modes TEM_{00} and TEM_{10}.

TABLE

Relative field distribution over the mirrors of a confocal resonator ($N=1$, $T<\alpha_0 d$)-TEM_{00} mode

$$r(\eta)=\frac{R_0(\eta)}{R_0(\eta=0)}$$

$\alpha_0 d$ [%] / $\eta=\frac{r}{a}$	0	2.5		5		7.5		10		15	
		$\xi=0$	$\xi=1$	$\xi=0$	$\xi=1$	$\xi=0$	$\xi=1$	$\xi=0$	$\xi=1$	$\xi=0$	$\xi=1$
0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	0.9743	0.9743	0.9742	0.9742	0.9742	0.9742	0.9742	0.9742	0.9741	0.9741	0.9740
0.2	0.9004	0.9003	0.9002	0.9003	0.9001	0.9002	0.8999	0.8999	0.8998	0.8997	0.8995
0.3	0.7876	0.7875	0.7873	0.7874	0.7870	0.7872	0.7867	0.7867	0.7864	0.7863	0.7859
0.4	0.6495	0.6494	0.6492	0.6492	0.6488	0.6489	0.6483	0.6481	0.6478	0.6475	0.6471
0.5	0.5019	0.5017	0.5014	0.5013	0.5009	0.5008	0.5003	0.5000	0.4998	0.4992	0.4989
0.6	0.3596	0.3593	0.3591	0.3589	0.3586	0.3582	0.3580	0.3575	0.3574	0.3565	0.3566
0.7	0.2349	0.2345	0.2345	0.2340	0.2340	0.2333	0.2334	0.2327	0.2329	0.2317	0.2323
0.8	0.1357	0.1352	0.1354	0.1346	0.1349	0.1339	0.1344	0.1337	0.1341	0.1327	0.1337
0.9	0.0649	0.0643	0.0646	0.0636	0.0643	0.0639	0.0639	0.0632	0.0638	0.0624	0.0637
1.0	0.0209	0.0203	0.0207	0.0196	0.0205	0.0191	0.0203	0.0197	0.0203	0.0191	0.0205

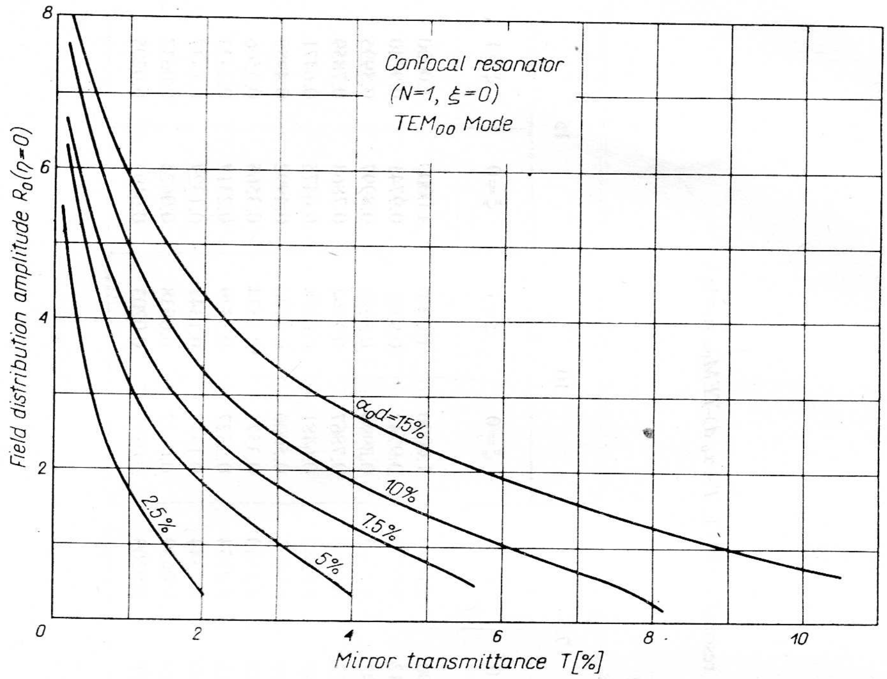

Fig. 3. Amplitude of the field distribution over the confocal resonator mirrors versus mirror transmittance for various values of the gain coefficient $\alpha_0 d$ of the medium ($\xi=0$)

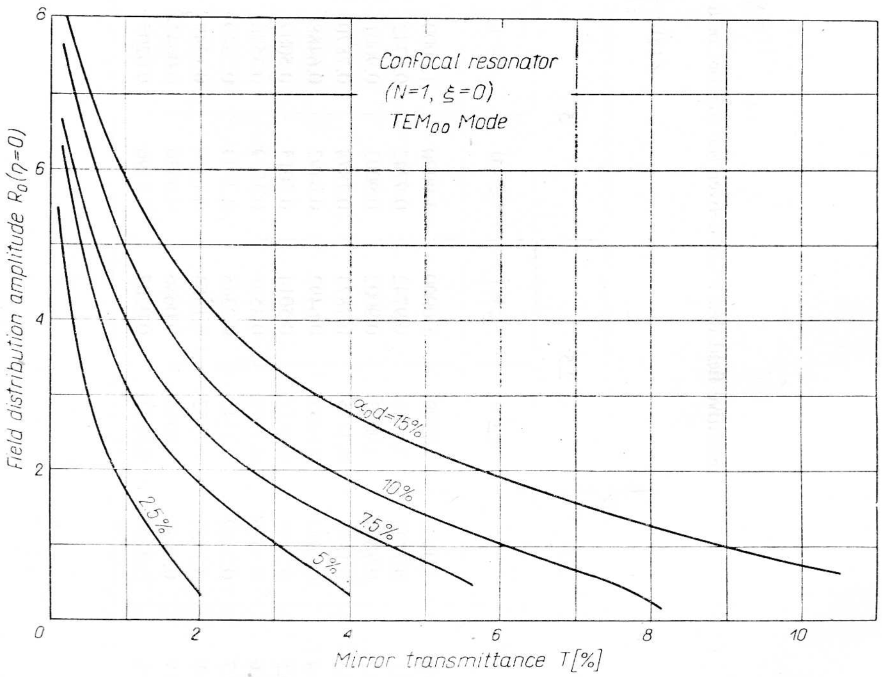

Fig. 4. Amplitude of the field distribution over the confocal resonator mirrors versus mirror transmittance for various values of the gain coefficient $\alpha_0 d$ of the medium ($\xi=1$)

The amplitude of the field distribution $R_0(\eta=0)$ versus mirror transmissivity T, for various $(\alpha_0 d)$ values and fixed $N=1$ and $\xi=0$ or 1, is shown in Figs. 3 and 4. The effective gain is, for the uniform gain distribution $(\xi=0)$ larger than for the parabolical gain distribution $(\xi=1)$. This results in larger field distribution amplitude in the former case comparing to the latter. Analogously, the power transmitted outside the resonator, which is proportional to the $R^2(\eta=0)\cdot T$ product, should be for the resonator filled with a medium with the uniform gain distribution higher than for that with the parabolical gain distribution, if only the amplitudes of both the gain distributions are equal. The power transmitted outside the confocal resonator was shown in Figs. 5 and 6 as a function of mirror transmittances, for various $\alpha_0 d$ values and fixed: $N=1$, $\xi=0$ or 1. It is evident that these figures confirm the above assumption.

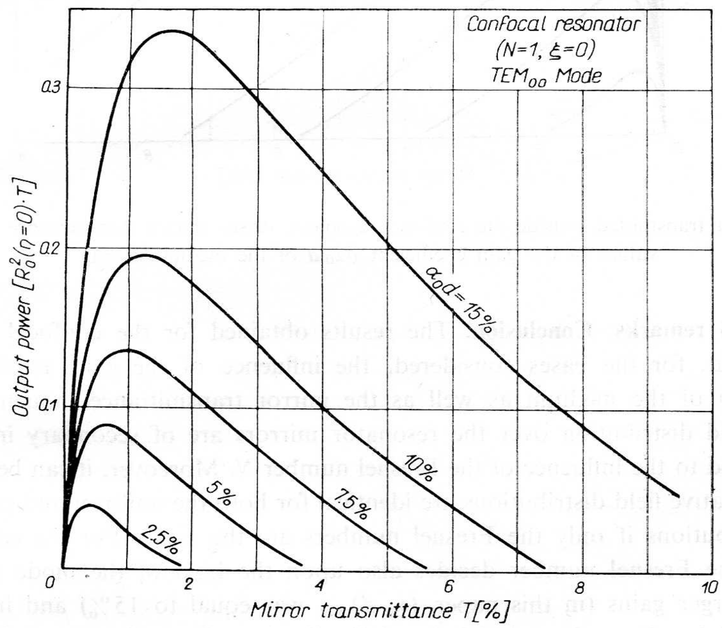

Fig. 5. Power transmitted outside the confocal resonator versus mirror transmittance for various values of the gain coefficient $\alpha_0 d$ of the medium $(\xi=0)$

The diagrams shown in Figs 5 and 6 serve for determination of mirror transmittance values optimum with regard to the power transmitted, for various gain values $\alpha_0 d$. The optimum value estimated for a medium with the uniform gain distribution $(\xi=0)$ equals to $T_{opt}=(0.13\pm0.01)\ \alpha_0 d$ for $\alpha_0 d$ within the limits (2.5—15) % whereas for parabolically distributed gain of the medium $(\xi=1)$ the optimum value is $T_{opt}=(0.12\pm0.01)\ \alpha_0 d$ for the same range of $\alpha_0 d$ variability. Thus, the optimum transmittances can be regarded in practice as independent of gain distributions if only their amplitudes are identical.

No systematic calculations for higher N were made because the higher is N the longer is time required for the highly time-consuming method of calculations adopted.

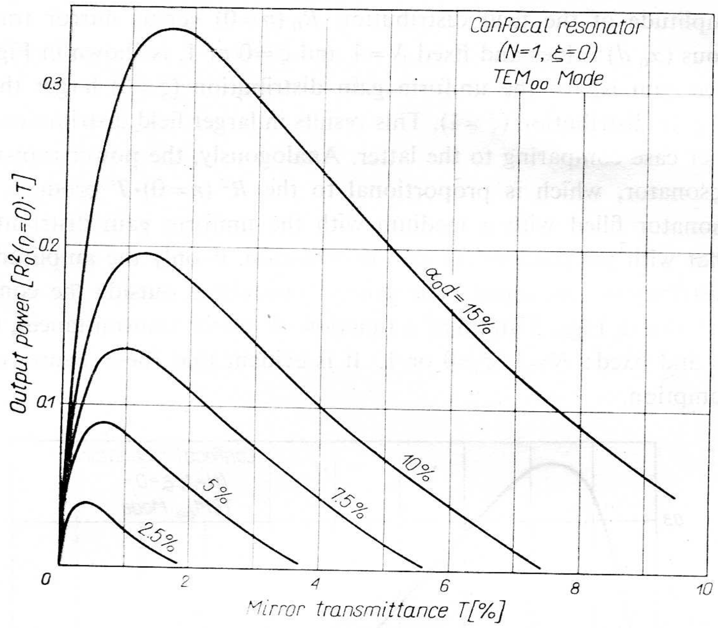

Fig. 6. Power transmitted outside the confocal resonator versus mirror transmittance for various values of the gain coefficient $\delta\alpha_0 d$ of the medium ($\xi=1$)

2. Final remarks. Conclusion. The results obtained for the confocal resonator showed that, for the cases considered, the influence of the gain amplitude and distribution of the medium as well as the mirror transmittance influence on the relative field distribution over the resonator mirrors are of secondary importance as compared to the influence of the Fresnel number N. Moreover, it can be admitted that the relative field distributions are identical for both the uniform and parabolical gain distributions if only the Fresnel numbers are the same. For the cases under question the Fresnel number decides also upon the kind of the mode generated. Yet, for larger gains (in this paper $(\alpha_0 d)_{max}$ was equal to 15%) and in the first place for higher N numbers the kind and distribution of a mode generated is supposedly affected by the shape of the gain distribution. However, calculations for higher values of $\alpha_0 d$ and N were not carried out, owing to the limited range of validity of the derivation of the resonator equation (21) ($\alpha_0 d \ll 1$) as well as due to highly time-consuming calculations for $N>1$.

The amplitude of a field distribution and the power transmitted outside the resonator are smaller for parabolical decrease of the gain along the radius as compared to the uniform gain distribution. The difference is, however, not so large as it could be expected basing on the estimated (e.g. from the mean value of the gain, obtained by overaging the parabolical distribution along paths perpendicular to the resonator axis) change of the effective gain of the medium. This suggests that those parts of the amplifying medium that are more distant from the resonator axis affect but slightly the generation of the TEM_{00} mode — most thoroughly analyzed