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PHENOMENOLOGICAL MODELS OF CAVITATION EROSION PROGRESS

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Abstract: *Numerical procedures determining parameters of the erosion curves according to the phenomenological models of J.Noskievič, K.Steller and L.Sitnik have been developed and tested. The method of Marquardt-Levenberg has proved especially suitable for this purpose. Basing on a comparative analysis, the models of K.Steller and L.Sitnik are recommended for more advanced applications. Numerical procedures have been tested using results obtained within the framework of the International Cavitation Erosion Test project.*

1. EROSION CURVES

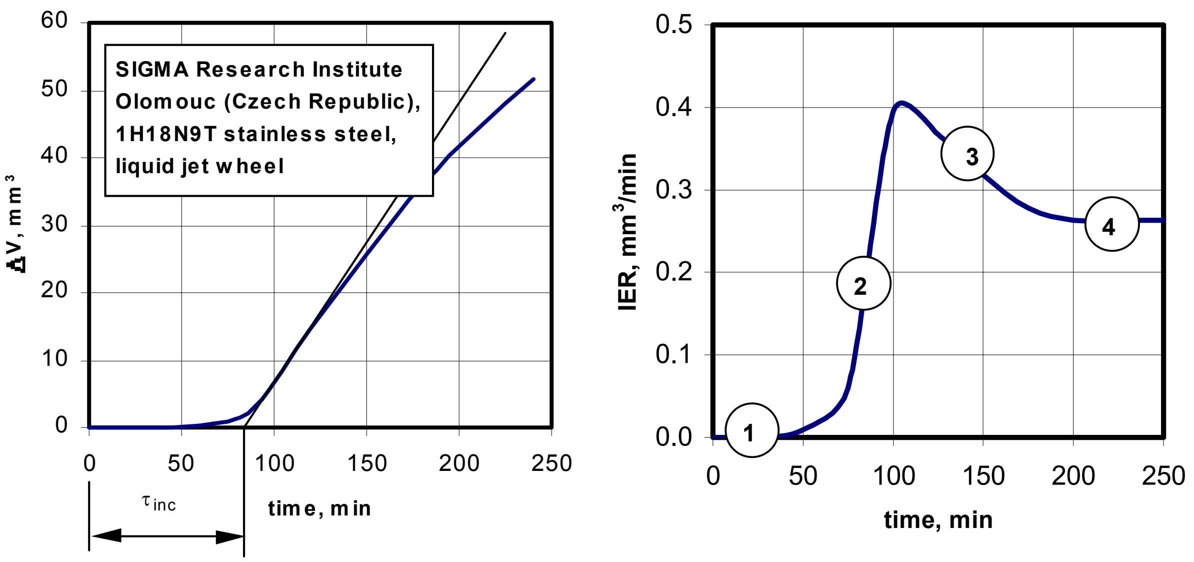

Fig.1 Typical erosion curves pattern and the definition of stipulated incubation period at an example of ICET results [1, 2]; 1 - incubation period, 2 - acceleration period, 3 - deceleration period, 4 - steady state erosion period

Typical damage progress in materials subjected to highly concentrated mechanical impingement (e.g. due to the impact of solid or liquid particles, or collapse of cavitation bubbles and vortices) can be divided into several characteristic periods (Fig.1):

- incubation period, in which internal stresses are cumulated in the surface layer without any visible volume losses,

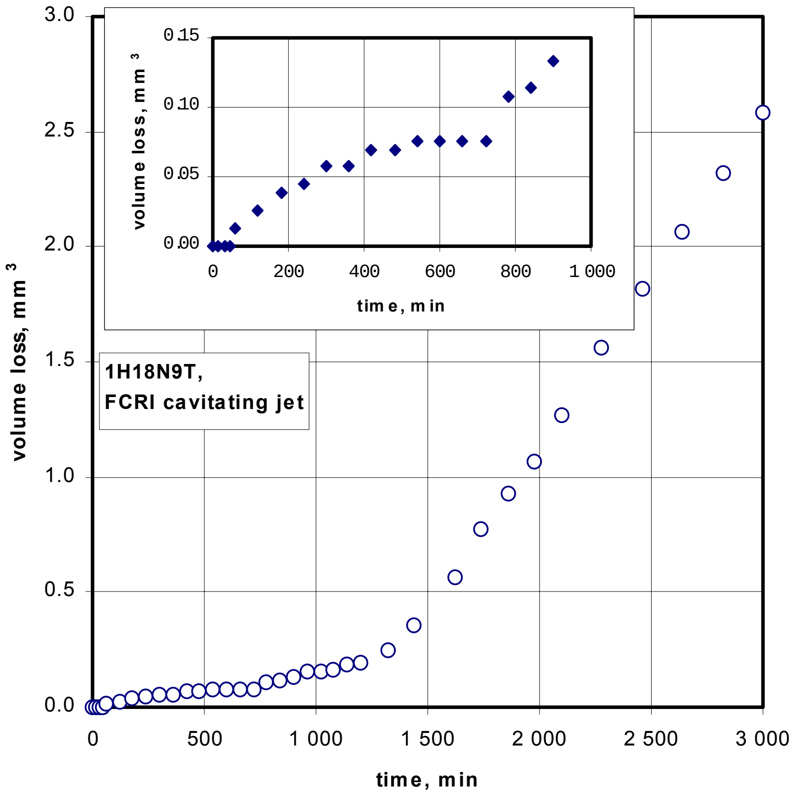

Fig.2 Erosion progress of 1H18N9T steel at the FCRI Lichtarowicz cell

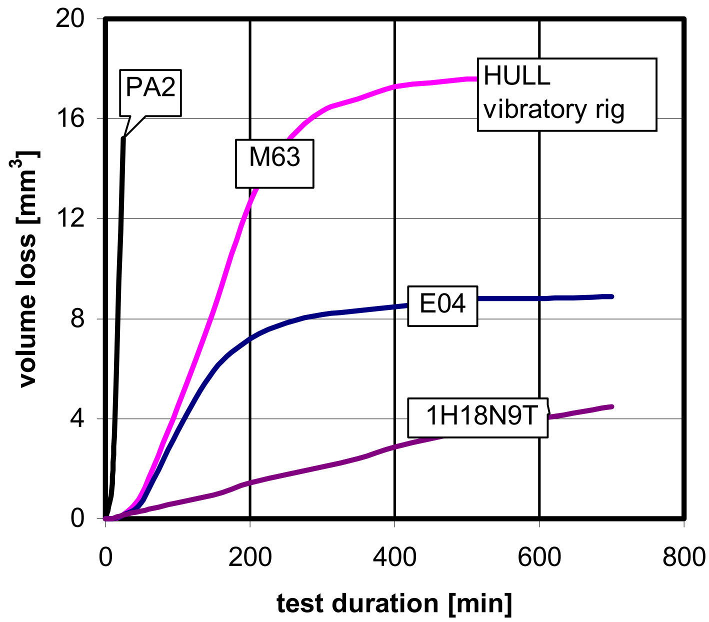

Fig.3 Erosion curves determined at the vibratory rig in the Hull University lab

- acceleration period resulting out of almost simultaneous exceeding the fatigue strength limit at a large surface area,
- deceleration period due to uncovering of major areas with no internal stresses cumulated so far,
- steady state damage period featured by a balance of energy cumulated and lost with removed material particles.

It should be noticed that this general schematic and especially the physical interpretation of individual damage periods, neglects several factors of substantial significance for the erosion progress. These factors include the following facts:

a) in case of low yield stress materials (e.g. numerous aluminium alloys), the mass loss can take place even in result of a short-time impingement and the fatigue effects are of minor significance which results in continuous rise of erosion rate, without detectable incubation period,

b) in case of a number of materials (especially those of metastable structure) the surface layer work-hardening effect takes place in the initial stage of damage which elongates the incubation period and decreases the erosion rate

c) the efficiency of cavitation erosive effects can be substantially affected by the surface state of the material: on the one hand side surface roughness can increase the damage rate in the initial stage of erosion, on the other hand side - surface waviness due to collective action of collapsing cavities can diminish the impingement intensity in advanced stages of the process (vibratory rigs)

d) change of material surface geometry due to cavitation damage can result in an increase of loading due to development of additional cavitators (especially at flow confining surfaces) or in a decrease of loading due to total destruction of material in the highly loaded area (e.g. during erosion tests at fluid-flow rigs or at vibratory rigs with stationary specimens)

e) in numerous cases the progress of mechanical erosion is accelerated by electrochemical phenomena (corrosion)

The ICET results [1, 2] include only few cases of classic erosion curve pattern (Fig.1). In most cases the steady state erosion period has not been attained which results out of an asymptotic fall of erosion rate. In some cases also the maximum Instantaneous Erosion Rate ($IER = \mathrm{d}(\Delta V)/\mathrm{d}t$) was not attained which seems due to negligible fatigue effects, too short test duration or significant variation of loading intensity due to the surface layer deformation. In some cases the test was carried so long that the period of decelerated damage was followed by subsequent increase of erosion rate (Lichtarowicz cell in the FCRI lab, Fig.2). In one case (vibratory rig with a stationary specimen in Hull University, Fig.3) the erosion rate fell to zero (most probably due to the total destruction of the eroded layer). It is obvious that tests carried out under variable loading conditions cannot be considered proper basis for assessment of erosion resistance of materials. Therefore, it is often recommended not to extend cavitation erosion tests beyond the point of maximum Cumulative Erosion Rate ($CER = \Delta V/t$).

All the ICET results were analysed basing on the manually plotted erosion curves. Such a procedure allowed avoiding oversimplifications possible in case of computer aided fitting to an improperly selected erosion progress model. The disadvantage of this procedure is certain ambiguity in determining the instantaneous erosion rate (IER) curves and such commonly used single-number parameters as the maximum instantaneous erosion rate (IER_{max}), maximum Mean Depth of Penetration Rate ($MDPR_{max}$), or the incubation period (τ_{inc})[1]. The obvious disadvantages of this procedure consist also in significant labour-consumption and complete lack of possibility to include them into more advanced algorithms [3]. These premises have decided on authors' interest in the previously developed phenomenological models of the erosion progress kinetics.

2. PHENOMENOLOGICAL MODELS OF CAVITATION EROSION PROGRESS

From among early papers on the subject, that presented by F.J.Heymann during a symposium held by the ASTM in 1966 [4] deserves particular attention. Assuming the eroded material to consist of subsequent layers of unit thickness and $f(t)\mathrm{d}t$ to be the probability of removing an element of each layer in the elementary time period [t, t+dt], the author shows that the erosion rate of material with unit surface area can be described by the integral equation

$$Y(t) = f(t) + \int_0^t f(t-T)Y(T)dT \tag{1}$$

[1] Therefore it is often recommended to use the cumulative (CER) instead of the instantaneous (IER) erosion rate parameter. The CER parameter is considered much less susceptible to ambiguity resulting from the manual plotting of the volume loss curves.

where the expression under the integral sign denotes erosion rate at the surface element uncovered in the instant *T*.

F.J.Heymann states that realistic erosion curves can be obtained by assuming the $f(t)$ function to vanish in the initial period of exposure ($t \leq T_0$), and then to be described by a three-parameter logarithmic-normal distribution

$$f(t) = \frac{1}{\sigma(t - T_0)\sqrt{2\pi}} \exp\left\{\frac{-[\ln(t - T_0) - m]^2}{2\sigma^2}\right\} \tag{2}$$

For the original surface layer F.J.Heymann proposes to use a distribution with parameters different from those applied for the subsequent layers. However, there are no hints on the method of their determination to be found in reference [4].

Later on, J.Noskievič [5] and K.Steller [6] proposed models of erosion kinetics referring directly to the erosion curve pattern.

J.Noskievič has noticed that the instantaneous erosion rate curve, *IER =IER(t)* reminds that of damped harmonic oscillator motion. The equation of motion of such an oscillator can be written as

$$\frac{d^2(IER)}{dt^2} + 2\alpha\frac{d(IER)}{dt} + \beta^2 IER = I\ . \tag{3}$$

with α and β denoting coefficients responsible for material properties (α - work-hardening capability, β - cavitation resistance), and I - parameter characterising intensity of cavitation erosive activity ($I = const$). By assuming erosion rate and its first derivative to vanish at the beginning of the process, the solution of equation (3) can be written down as

$$IER(t) = \begin{cases} v_s + v_s \dfrac{e^{-\alpha t}}{2\kappa}\left[(\alpha-\kappa)e^{-\kappa t} - (\alpha+\kappa)e^{\kappa t}\right] & \text{if } \ |\alpha| > \beta \\ v_s - v_s(1+\alpha_2 t)e^{-\alpha t} & \text{if } \ \alpha = \pm\beta \\ v_s - v_s\left(\cos\omega t - \dfrac{\alpha}{\omega}\sin\omega t\right)e^{-\alpha t} & \text{if } \ |\alpha| < \beta \ \text{ and } \ \alpha \neq 0 \\ v_s - v_s\cos\omega t & \text{if } \ \alpha = 0 \end{cases}, \tag{4}$$

where $\kappa = \sqrt{\alpha^2 - \beta^2}$ and $\omega = \sqrt{\beta^2 - \alpha^2}$, whence the volume loss curve is given by:

$$\Delta V = v_s\left[t - 2\frac{\alpha}{\beta^2} + \frac{e^{-\alpha t}}{2\kappa}\left(\frac{\alpha+\kappa}{\alpha-\kappa}e^{kt} - \frac{\alpha-\kappa}{\alpha+\kappa}e^{-kt}\right)\right] \qquad \text{if } |\alpha| > \beta \tag{5a}$$

$$\Delta V = v_s\left[t - \frac{2}{\alpha} + \left(t + \frac{2}{\alpha}\right)e^{-\alpha t}\right] \qquad \text{if } \alpha = \pm\beta \tag{5b}$$

$$\Delta V = v_s\left\{t - 2\frac{\alpha}{\beta^2} + \frac{\alpha}{\beta^2}e^{-\alpha t}\left[\left(\frac{\alpha}{\omega} - \frac{\omega}{\alpha}\right)\sin\omega t + 2\cos\omega t\right]\right\}, \qquad \text{if } |\alpha| < \beta \text{ and } \alpha \neq 0 \tag{5c}$$

$$\Delta V = v_s\left(t - \frac{1}{\omega}\sin\omega t\right), \qquad \text{if } \alpha = 0 \tag{5d}$$

The formula proposed by K.Steller is based on the notion of cavitation resistance R_{cav}, clearly linked to that of A.P.Thiruvengadam's erosion strength [7] and defined by the equation:

$$Pt = R_{cav} \Delta V \tag{6}$$

where P is the power used to erode the material subject to cavitation impingement. Cavitation is assumed a continuous process (proceeding at variable speed) resulting in the change of both material surface and its internal structure, responsible for cavitation resistance. Cavitation resistance dependence on duration of cavitation impingement is assumed to be described by an exponentially decreasing function

$$R_{cav} = \frac{\chi + e^{-\kappa t}}{\chi + 1} R_0 \tag{7}$$

The formula derived

$$\Delta V = \frac{(1+\chi)Pt}{R_0\left(\chi + e^{-\kappa t}\right)} \approx \frac{2(1+\chi)EPt}{\sigma^2\left(\chi + e^{-\kappa t}\right)} \tag{8}$$

shows clear dependence of the volume loss curve $\Delta V = \Delta V(t)$ on some basic material parameters (modulus of elasticity E and ultimate stress values σ), χ factor determining the ratio of ultimate and initial cavitation resistance (equation (7)), κ parameter defining the speed of cavitation resistance decrease in course of the process and power P used to erode the material.

In mid eighties L.Sitnik [8] assumed the time interval Δt between removal of the consecutive particles from the material surface to be a random function with a three-parameter logarithmic-natural distribution function

$$F_p(\Delta t) = 1 - \exp\left\{-\alpha_p\left[\ln(\Delta t/\Delta t_0 + 1)\right]^{\beta_p}\right\}. \tag{9}$$

The volume loss curve derived takes the form

$$\Delta V = \alpha_p \Delta V_0 \left[\ln(t/\Delta t_0 + 1)\right]^{\beta_p} \tag{10}$$

with ΔV_0 denoting certain characteristic volume value. The α_p parameter is characteristic for the intensity of erosive action of the phenomenon. It is worthwhile to notice that the model is a simplified version of the F.J.Heymann model.

3. DETERMINING THE VOLUME LOSS CURVE PARAMETERS ACCORDING TO THE SELECTED MODELS OF CAVITATION EROSION KINETICS

The least squares method has been adopted in order to determine the volume loss parameters according to selected models. The task can be reduced to that of minimising the expression

$$\chi^2(\boldsymbol{k}) = \sum_j \alpha_j \left[\Delta V_j - U(\boldsymbol{k}, t_j)\right]^2, \tag{11}$$

with α_j denoting the weight coefficient, $U(\, . \, , t)$ - a function describing erosion progress according to the model selected, and $\boldsymbol{k}$ - a set (vector) of parameters to be determined by means of the minimising procedure.

During calculations of that kind particular attention should be brought to an appropriate selection of the zero-th approximation.

In case of the J.Noskievič model the $\alpha = \pm\beta$ or $\alpha = 0$ condition is assumed in the zero-th approximation. Developing expressions (5b) and (5d) into a Mc Laurin series yields:

$$U(\boldsymbol{k},t) \approx u_s\left(\frac{\alpha^2 t^3}{6} - \frac{\alpha^3 t^4}{12}\right) = at^3 + bt^4 \qquad \text{if } \alpha = \pm\beta \tag{12a}$$

$$U(\boldsymbol{k},t) = u_s\left(\frac{1}{6}\omega^2 t^3 - \frac{1}{120}\omega^4 t^5 + \ldots\right) \approx at^3 + bt^5 \qquad \text{if } \alpha = 0 \tag{12b}$$

Parameters occurring in the formulae above can be easily determined by means of a linear regression. The formula resulting in the smaller value of the χ^2 parameter is selected as zero-th approximation for the minimising procedures.

In case of the K.Steller model the $\chi = 0$ condition is assumed in the zero-th approximation and parameter κ is determined from the approximate formula [7]

$$\kappa = \frac{3}{T}\frac{\Delta V_T T - 2\int_0^T \Delta V dt}{\Delta V_T T}, \tag{13}$$

with T denoting the total exposure duration of the material, and ΔV_T – total volume loss corresponding to this period. Sufficient accuracy is attained when using the trapezoid method of integration. The geometrical interpretation of expression (13) is explained in reference [7].

In case of the L.Sitnik model $\Delta t_0 = \tau_{inc}$ condition is assumed in the zero-th approximation while α and β parameters are determined from the linear regression of the

$$\ln U = a + b\ln\left[\ln\left(t/\Delta t_0 + 1\right)\right] \tag{14}$$

function with $a = \ln\alpha$ and $b = \beta$. The zero-th approximation of the $\Delta t_0 = \tau_{inc}$ parameter is determined by approximating the volume loss curve with the (12a) binomial.

The following minimisation procedures described in the excellent monograph by W.H.Press et al. [8] have been selected:

- the simplex method (comparison of values at the tops of a simplex constructed in the space of parameters, followed by its reflections, contractions and elongations in the direction determined by the minimum value of the minimised function)
- the Powell method (a non-gradient minimisation method along the basic vectors with configuration modified in subsequent steps)
- the Levenberg-Marquardt (combination of the steepest slope method with that of reversed hessian by scaling the diagonal of the Hessian matrix)

Application of some other methods, including those of the steepest slope, conjugate gradients in the Fletcher-Reeves or Polak-Ribier version, reversed Hessian (generalised Newton method) and the BFGS method of variable metrics was also considered in the initial stage of developing the appropriate numerical code.

4. NUMERICAL CODE TO CALCULATE PARAMETERS OF EROSION CURVES AND THE ANALYSIS OF OBTAINED RESULTS

A computer code *APROX* [9] has been developed in the *Borland Pascal* language in order to accomplish the necessary calculations. The code enables approximation of mass, volume and mean depth of penetration curves following the models described in section 2, using one of the above mentioned minimisation methods. The curves of instantaneous and cumulative erosion rate and basic single-number parameters are also determined. In case of several specimens the code determines also the averaged curves and parameters. A hard copy of the computer screen, taken during processing of the E04 data at the IMP PAN rotating disk can be seen in Fig.4.

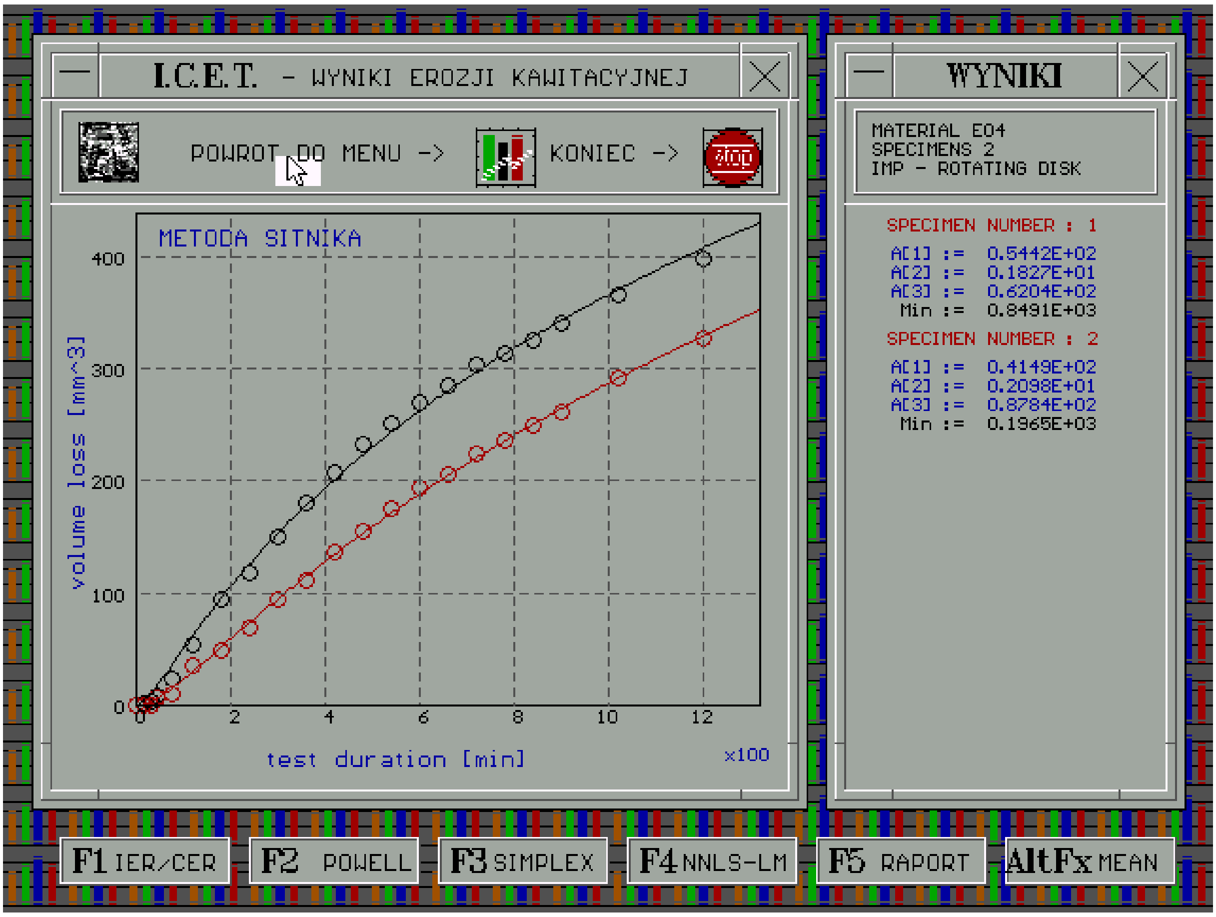

Fig.4 A hard copy of the computer screen, taken during processing the E04 data

The analysis conducted has proved very good convergence of all the methods in case the process under consideration covers also the period of steady or quasi-steady damage rate (Fig.5a).

The evident superiority of the simplex and Levenberg-Marquardt methods over that of Powell was stated already in the early stage of analysis. Currently, no problems occur when calculating the erosion curves according to the models of K.Steller and L.Sitnik. Calculation by means of the simplex method does not always end in global minimisation of formula (11).

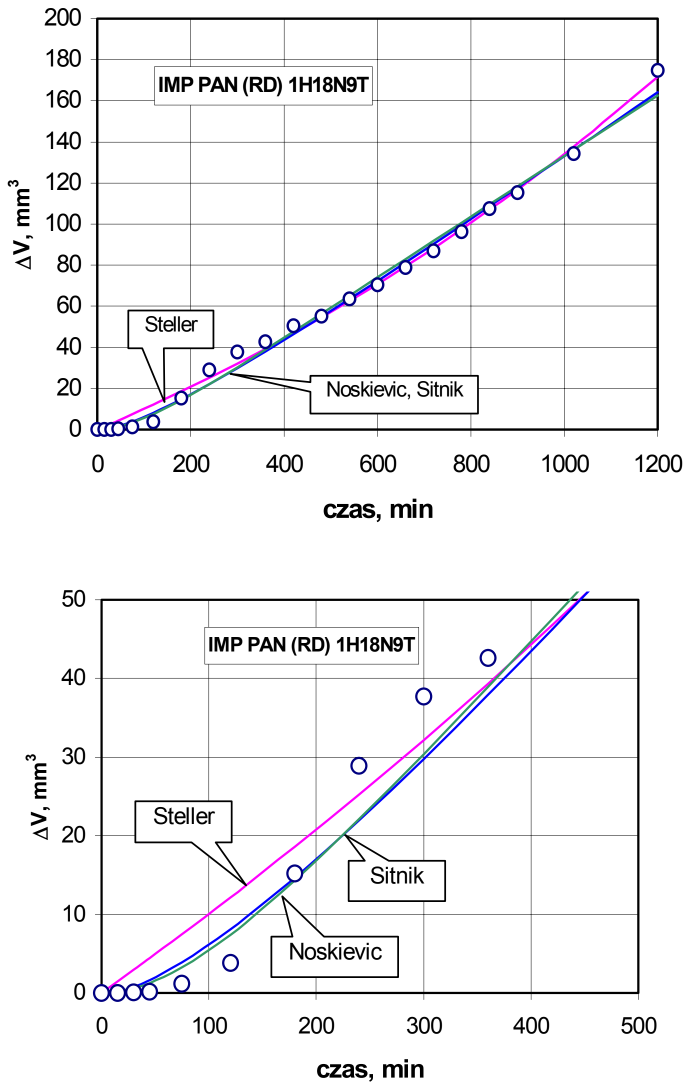

Fig.5 Erosion curves obtained at the IMP PAN rotating disk facility (the exposure period of 20 h duration is analysed)

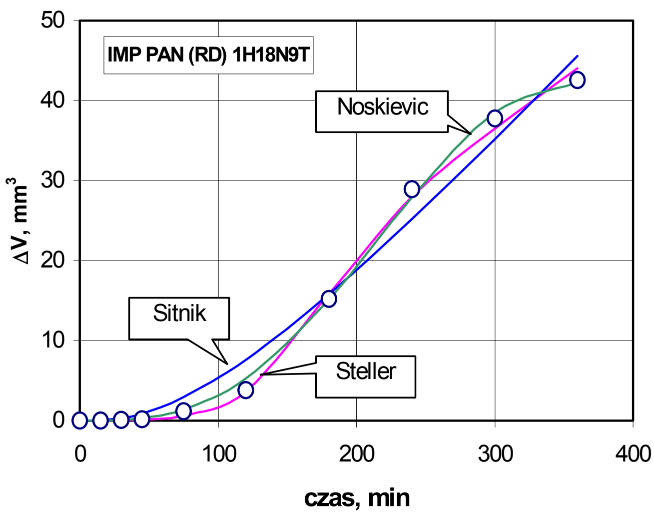

Fig.6 Erosion curves obtained at the IMP PAN rotating disk facility (the exposure period of 6 h duration is analysed)

The work on ensuring full reliability of calculations according to the J.Noskievič model is now in progress and is expected to end successfully in a short time. Nevertheless the authors are reluctant to recommend it as an element of more sophisticated algorithms due to a rather complicated form of formula (5), incorporating additionally some periodical functions.

No numerical problems have been encountered when testing the model of K.Steller. However, one should notice the non-vanishing first derivative of formula (7) at the beginning of the process, which can result in improper modelling of the incubation period and improper assessment of the maximum instantaneous erosion rate (Fig.5b). On the other hand side, if the analysis is confined to the period preceding attainment of the maximum cumulative erosion rate (CER_{max}), the model can describe the reality even better than the other ones (Fig.6)

Following the model of L.Sitnik, the erosion rate in the initial stage of damage is always equal zero. The curves obtained show a very smooth pattern, hardly dependent on the duration of analysed period. Therefore, under some circumstances, the model can describe material performance in the initial damage period worse than the other ones. An obvious advantage is the very fact that the model has been developed basing on considerations of the stochastic nature of the cavitation erosion phenomenon. Thus, the model is a continuation of the original concept of F.J.Heymann, somehow related to the contemporary methods of modelling the fatigue phenomena [11].

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