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A CONCEPT OF FRACTIONAL CAVITATION EROSION RESISTANCE

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Abstract: *A new method of predicting material performance under prescribed cavitation impingement conditions is proposed. The method assumes the volume loss due to a polyfractional material impingement by cavitation micro- and macropulses to be a superposition of volume losses due to individual fractions. Volume loss due each fraction is considered to be an analytical function of the energy flux delivered to the surface. Material resistance to each fraction is defined by a set (single-fractional vector) of parameters of this curve. Material resistance to the realistic (polyfractional) impingement is described by a matrix composed of single-fractional vectors.*

1. CAVITATION PULSES DISTRIBUTION AND THE DAMAGE RATE

The concept of linking the volume loss ΔV with the energy E_a absorbed by eroded material is due to A.P.Thiruvengadam [1] who used the formula

$$E_a = \Delta V S_e \tag{1}$$

in order to define the erosion strength parameter S_e.

Later on, it was realised that the energy delivered in a time unit to the impinged surface unit by a collapsing cavity can be considered proportional to the well-known expression

$$I = \frac{p^2}{\rho c} \tag{2}$$

where p denotes the maximum pressure at the surface while ρ and c are liquid density and sound celerity, respectively. By considering cavitation impingement of a unit surface to be described by a Poisson process, J.Kirejczyk [2] showed in 1979 that the total energy flux delivered per time unit to a unit surface of impinged material can be considered proportional to the expression

$$ME = \frac{\varepsilon}{\rho c}\frac{\lambda}{N}\sum_{i=1}^{N} n_i p_i^2 = \frac{k}{\rho c}\sum_{i=1}^{N} n_i p_i^2 \tag{3}$$

parameter with n_i denoting the number of pressure pulses of amplitude p_i measured per time unit, λ - the Poisson process factor and ε - a proportionality coefficient assumed by J.Kirejczyk to be close to 5×10^{-5} s[1]. In the later papers, e.g. [3], good correlation between the ME cavitation intensity parameter and the erosion rate of aluminium and zinc specimens in the initial period of damage has been proved in the Institute of Fluid-Flow Machinery of the Polish Academy of Sciences (IMP PAN) lab. Reports of other authors are also considered encouraging.

[1] The constant $k = 10^{-6}$ s is used instead

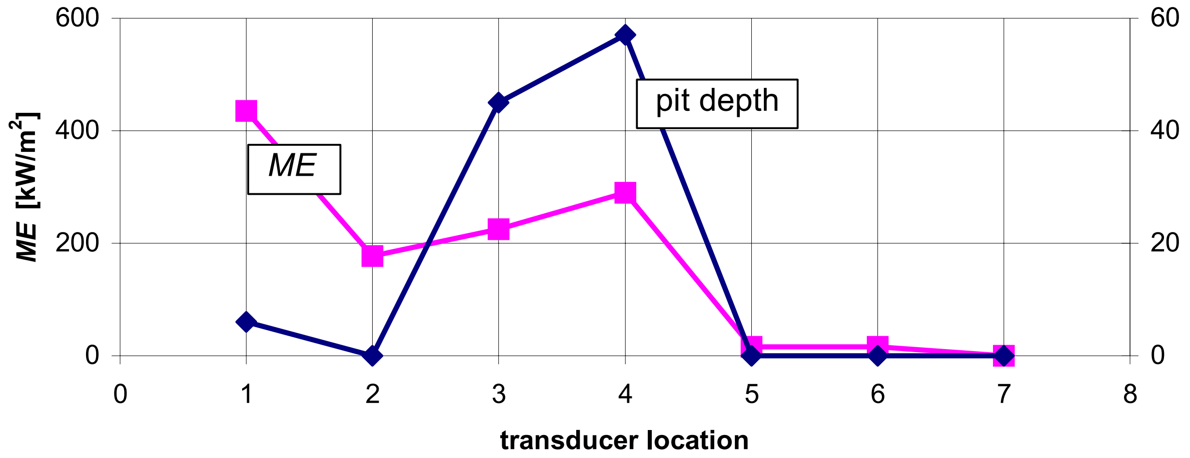

Fig.1 Distribution of the mean depth of erosion pits *h* and the index of the energy flux density *ME* [kW/m^2] measured along a zinc specimen in the IMP PAN cavitation tunnel [5]

Despite of these positive results, it was felt right from the beginning that the assumption of direct proportionality between the *ME* cavitation intensity parameter and the volume loss rate may fail even in the initial period of erosion if qualitatively different distribution of cavitation pulses are applied. This anticipation was confirmed by the results obtained in an IMP PAN cavitation tunnel with two cylindrical barricades [4]. A different distribution of the *ME* parameter and the erosion depth along zinc specimen situated downstream of the slot cavitator (Fig.1) was explained by high threshold corresponding to initiation of plastic deformation in the specimen. Later on, it was shown that a significant amount of low energy pulses results from macroscopic pressure fluctuations, which can be detected using a pair of piezoelectric transducers situated at the same distance from the slot. Tests conducted in 1997 in a modified test chamber, resembling that of Erdmann-Jesnitzer (Fig.2) prove that the macroscopic pressure pulses acting simultaneously on a large surface should not be neglected from consideration. Experimental results obtained so far show that a change of the ratio between macro and micro pulses results both in the change of volume loss curve and the pattern of erosion at the impinged surface of a zinc specimen [5].

It should be noticed that the ME parameter was used so far assuming only one type of pulses. In fact, in order to take account of both micro and macro pulses and make the measurement independent of the transducer membrane size, one should replace formula (3) with the formula

$$ME = C \cdot \left(\frac{A}{A_0} \sum_i n_i p_i^{\,2} + \sum_i N_i P_i^2 \right) \tag{4}$$

where small letters have been reserved for the micro pulses acting only on a small fraction of the membrane surface while capitalics are used to describe the macro pulses due to collective phenomena acting on the whole membrane surface. A and A_0 symbols are used for the membrane surface of the current and reference transducer, respectively. The method of distinguishing between the micro and macro pulses is based on criterion of time shift between peaks identified at two transducers. Appropriate algorithm is described in [6]. A phenomenological model allowing to describe erosion progress due impingement consisting of both the micro and macro pulses is discussed in the next section.

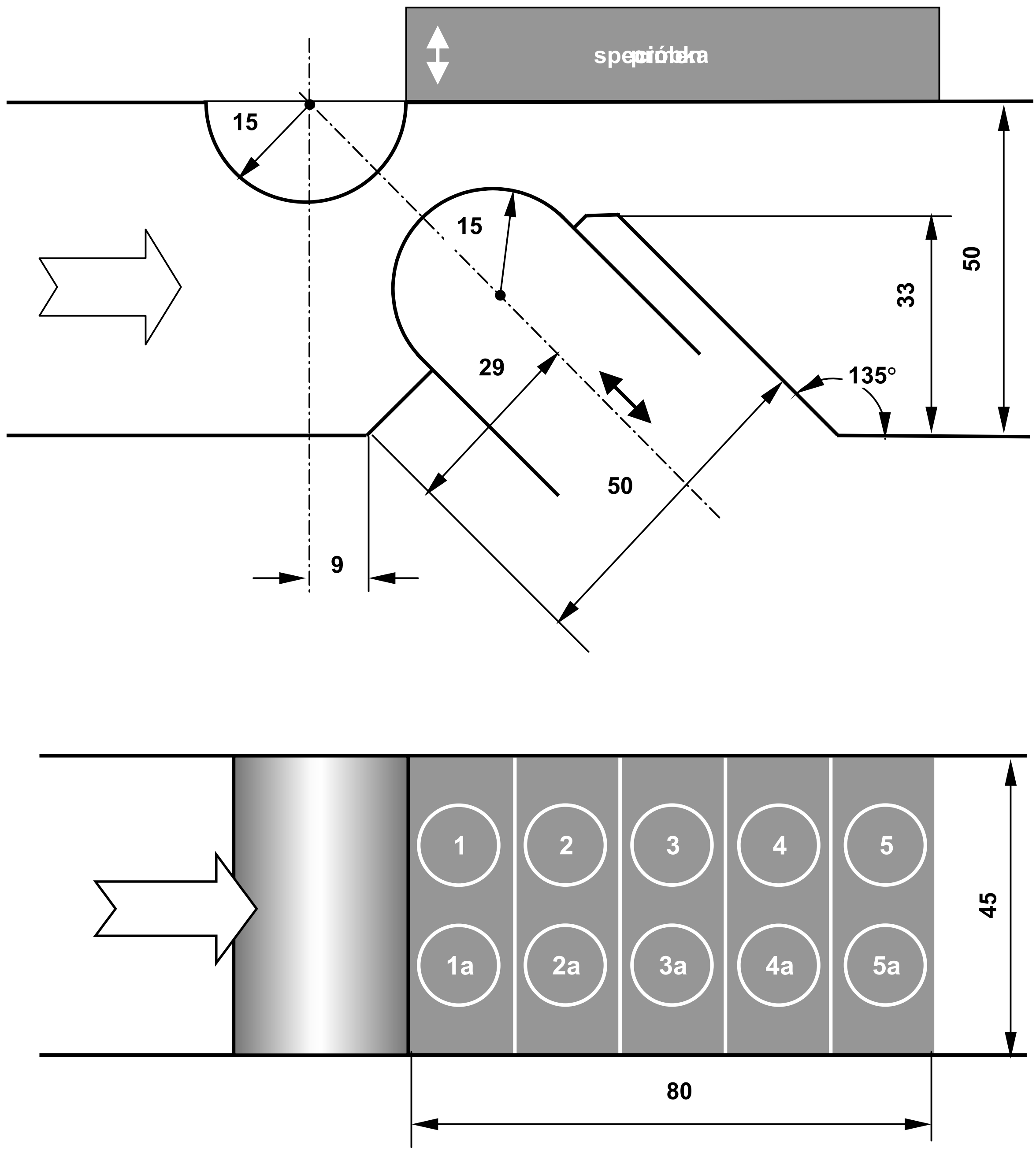

Fig.2 Schematic of the Erdmann-Jesnitzer test chamber in the IMP PAN lab with position of piezoelectric pressure transducers indicated (all dimensions are in mm)

2. POLYFRACTIONAL CAVITATION EROSION MODEL

The volume loss curve is usually described by the formula

$$\Delta V = A\Im \cdot U(\boldsymbol{k}, t) \tag{5}$$

with A denoting the eroded surface area, $\Im$ - a measure of cavitation intensity, U – erosion progress function resulting out of applied phenomenological model, $\boldsymbol{k}$ – a set of parameters determined by fitting the erosion curve to the experimental data (usually 3 parameters are quite sufficient), t – cumulative exposure duration

There exists a number of phenomenological models describing cavitation erosion progress, just to mention those of J.Noskievič [7], K.Steller [8] and L.Sitnik [9].

Following J.Noskievič, a volume loss curve can be described by the function

$$\Delta V = v_s t + \frac{v_s}{2\kappa}\frac{\alpha+\kappa}{\alpha-\kappa}\left[e^{-(\alpha-\kappa)t}-1\right] - \frac{v_s}{2\kappa}\frac{\alpha-\kappa}{\alpha+\kappa}\left[e^{-(\alpha+\kappa)t}-1\right] = v_s\left[t - 2\frac{\alpha}{\beta^2} + \frac{e^{-\alpha t}}{2\kappa}\left(\frac{\alpha+\kappa}{\alpha-\kappa}e^{kt} - \frac{\alpha-\kappa}{\alpha+\kappa}e^{-kt}\right)\right] \quad \text{if } |\alpha| > \beta \qquad (6a)$$

$$\Delta V = v_s\left[t - \frac{2}{\alpha} + \left(t + \frac{2}{\alpha}\right)e^{-\alpha t}\right] \quad \text{if } \alpha = \pm\beta \qquad (6b)$$

$$\Delta V = v_s\left\{t - 2\frac{\alpha}{\beta^2} + \frac{\alpha}{\beta^2}e^{-\alpha t}\left[\left(\frac{\alpha}{\omega} - \frac{\omega}{\alpha}\right)\sin\omega t + 2\cos\omega t\right]\right\}, \quad \text{if } |\alpha| < \beta \text{ and } \alpha \neq 0 \qquad (6c)$$

$$\Delta V = v_s\left(t - \frac{1}{\omega}\sin\omega t\right), \quad \text{if } \alpha = 0 \qquad (6d)$$

obtained by solving the differential equation

$$\frac{d^2 v}{dt^2} + 2\alpha\frac{dv}{dt} + \beta^2 v = I, \qquad (7)$$

with $v = d(\Delta V)/dt$ denoting the volume loss rate, α and β - coefficients determining material properties (work hardening ability and resistance to cavitation, respectively), I – cavitation intensity parameter ($I = const$), and v_s - the ultimate value of the volume loss rate, $v_s = I/\beta^2$. Prediction of erosion progress requires the knowledge of the characteristic material parameters (α and β) and the cavitation intensity parameter I.

The erosion model proposed by Steller [20] is based on the assumption that material resistance to cavitation, defined by the formula

$$R_{cav}\Delta V = Pt \qquad (8)$$

where P is mean power absorbed by the eroded material, diminishes according to the exponential law

$$R_{cav} = \frac{\chi + e^{-\kappa t}}{\chi + 1}R_0 \qquad (9)$$

The volume loss formula

$$\Delta V = \frac{2(1+\chi)Pt}{R_0\left(\chi + e^{-\kappa t}\right)} \approx \frac{2(1+\chi)EPt}{\sigma^2\left(\chi + e^{-\kappa t}\right)} \qquad (10)$$

shows ΔV dependence on the material properties (Young modulus E and tensile strength σ), χ coefficient describing the ratio of the ultimate and initial cavitation resistance, κ coefficient defining the rate of erosion resistance variation and the power P used to erode the material under investigation.

The author of the third formula assumed the time gap Δt between loosening of subsequent particles from the material surface to be a stochastic variable with a 3-parameter distribution function of the logarithmic-normal type

$$F_p(\Delta t) = 1 - \exp\left\{-\alpha_p\left[\ln\left(\Delta t/\Delta t_0 + 1\right)\right]^{\beta_p}\right\}. \quad (11)$$

The resulting volume loss curve takes the form

$$\Delta V = \alpha_p \Delta V_0\left[\ln\left(t/\Delta t_0 + 1\right)\right]^{\beta_p}, \quad (12)$$

where ΔV_0 is certain characteristic volume, α_p and β_p parameters describe the intensity of erosive cavitation attack, and Δt_0 is closely linked with the incubation period duration Periods of erosion incubation, acceleration and deceleration are easily discernible from the curves plotted using formula (12).

The general weakpoint of formula (5) and all the above mentioned erosion models is the use of the cumulative exposure time instead of energy delivered to the impinged surface as an independent variable. One can easily notice that in case of a uniform distribution of cavitation pulses this leads to lack of incubation time dependence on the number of pulses in a time unit. In order to avoid this paradox, formula (5) should be replaced by the formula

$$\Delta V = A \cdot U(\boldsymbol{k}, \Im t) \quad (13)$$

and formula (6) with the formula

$$\frac{1}{P^2}\frac{d^2 v}{dt^2} + \frac{2\alpha}{P}\frac{dv}{dt} + \beta^2 v = I = \gamma P, \quad (14)$$

which results in the volume loss curve

$$\Delta V = u_s\left[Pt - 2\frac{\alpha}{\beta^2} + \frac{e^{-\alpha Pt}}{2\kappa}\left(\frac{\alpha+\kappa}{\alpha-\kappa}e^{kPt} - \frac{\alpha-\kappa}{\alpha+\kappa}e^{-kPt}\right)\right] \quad \text{if } |\alpha| > \beta \quad (15a)$$

$$\Delta V = u_s\left[Pt - \frac{2}{\alpha} + \left(Pt + \frac{2}{\alpha}\right)e^{-\alpha Pt}\right] \quad \text{if } \alpha = \pm\beta \quad (15b)$$

$$\Delta V = u_s\left\{Pt - 2\frac{\alpha}{\beta^2} + \frac{\alpha}{\beta^2}e^{-\alpha Pt}\left[\left(\frac{\alpha}{\omega} - \frac{\omega}{\alpha}\right)\sin\omega Pt + 2\cos\omega Pt\right]\right\}, \quad \text{if } |\alpha| < \beta \text{ and } \alpha \neq 0 \quad (15c)$$

$$\Delta V = u_s\left(Pt - \frac{1}{\omega}\sin\omega Pt\right) \quad \text{if } \alpha = 0 \quad (15d)$$

where P denotes the power of energy flux delivered by the cavitation cloud to the eroded material, and γ is the coefficient defined by equation (14).

Consequently, formula (10) should be replaced with the formula

$$\Delta V = \frac{2(1+\chi)Pt}{R_0\left(\chi + e^{-\chi Pt}\right)} \approx \frac{2(1+\chi)EPt}{\sigma^2\left(\chi + e^{-\chi Pt}\right)} \quad (16)$$

and formula (12) – with the formula

$$\Delta V = \alpha_p \Delta V_0 \left[\ln\left(\frac{Pt}{\Delta E_0} + 1 \right) \right]^{\beta_p} \tag{17}$$

where $\Delta E_0 = P\Delta t_0$ is the energy amount related to the incubation period.

Equations (15÷17) can be resolved to the form (13) with a three-component vector $\boldsymbol{k}$ to be determined by fitting the theoretical curve to the experimental data. The *ME* cavitation intensity parameter is proportional to the power flux delivered to the impinged surface and can be used instead of the realistic *P* value without any loss of fitting accuracy.

However, in order to take account of the different material resistance to different classes and amplitude fractions of the cavitation load, the scalar parameter *ME* should be replaced by the

$$\boldsymbol{ME} = \begin{pmatrix} ME_1 \\ ME_2 \\ \dots \\ ME_{2n} \end{pmatrix} \tag{18}$$

vector where

$$ME_i = \begin{cases} C\dfrac{A}{A_0} n_i p_i^{\,2} & \text{for} \quad i = 1,\dots,n \\ CN_{i-n} P_{i-n}^{\,2} & \text{for} \quad i = n+1,\dots,2n \end{cases}$$

and formula (13) should be replaced by the superposition law

$$\Delta V = A \sum_{i=1}^{2n} U(\boldsymbol{k}_i, ME_i \cdot t) \tag{19}$$

where *n* denotes the number of fractions of micro and macro scale pulses and

$$\mathbf{K} = \begin{pmatrix} \boldsymbol{k_1} \\ \boldsymbol{k_2} \\ \dots \\ \boldsymbol{k_{2n}} \end{pmatrix} = \begin{pmatrix} k_{1,1} & k_{1,2} & k_{1,3} \\ k_{2,1} & k_{2,2} & k_{2,3} \\ \dots & \dots & \dots \\ k_{2n,1} & k_{2n,2} & k_{2n,3} \end{pmatrix}$$

- the matrix of erosion curves parameters describing material resistance to the individual classes and amplitude fractions of cavitation load.

Although it is recommended to divide whole the amplitude range into $n = 4$ classes, the previously mentioned experimental results allow to expect substantially better description of correlation between cavitation intensity and the erosion rate even if a two-component ($n = 1$) ***ME*** vector is used.

The obvious disadvantage of the superposition law (19) is asynchronous addition of erosion curves corresponding to individual amplitude fractions which can lead to unrealistic results. Therefore the method of instantaneous volume loss (erosion depth) increments developed by Weigle and Szprengiel [10] and explained in Fig.3 is used instead. The method assumes that the eroded material is “unaware” which load fraction s have contributed to the energy accumulation and volume loss having taken place since the erosion process has started. Therefore, instantaneous contribution of each fraction can be calculated as if only the currently considered fraction were responsible for whole the damage. The corresponding superposition law can be written down in form of a differential equation

$$\frac{d(\Delta V)}{dt} = A\sum_{i=1}^{2n} ME_i \frac{\partial U}{\partial E}(\boldsymbol{k}_i, E)\bigg|_{E=\Theta(\boldsymbol{k}_i, \Delta V/A)} , \tag{20}$$

with Θ standing for the function reverse to $U(\boldsymbol{k}_i, E)$ respective the second variable. The value of this function in formula (20) is the energy the monofractional cavitation impingement field should deliver to the material surface in order to erode it up to the mean depth $\Delta V/A$.

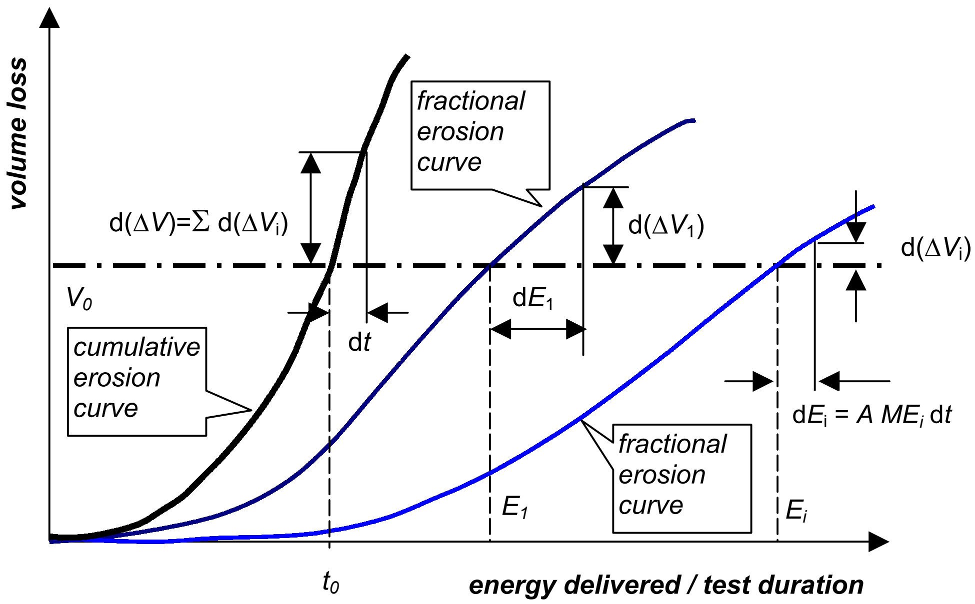

Fig.3 The principle of erosion curves superposition by means of the instantaneous volume loss increments [10]

Assuming the U in the shape proposed by Sitnik, that is

$$U = \alpha_p \left[\ln\left(Pt/\Delta E_0 + 1\right)\right]^{\beta_p} , \tag{21}$$

where α_p, β_p and ΔE_0 are components of the $\boldsymbol{k_i}$ vector, one can easily show that

$$\Theta = \frac{\Delta E_0}{\beta ME_i}\left[\exp\left(\Delta V/\alpha A\right) - 1\right] \tag{22}$$

When the U function in the shape proposed by Noskievič or Steller is used, numerical procedures have to be applied in order to calculate the Θ function value. Numerical procedures are also needed in order to calculate the

$$\Delta V = A \cdot \mathrm{CEF}\left(\mathbf{K}, \boldsymbol{ME}, t\right) \tag{23}$$

value where the CEF (*Cumulative Erosion Function*) symbol is used for the solution of differential equation (20) respective $\Delta V/A$ variable.

Due to the shape of equation (20), the *predictor-corrector* method seems to be suited for the aforesaid purpose. The Θ function can be found using the *regula falsi* method. The method of determining the components of the ***ME*** vector and the **K** matrix is discussed in the next section.

3. NEW METHOD OF CAVITATION RESISTANCE ASSESSMENT

The method proposed is based on the following assumptions:

1. The volume loss curves $\Delta V = \Delta V(t)$ of materials subjected to cavitation impingement are superposition of the curves corresponding to individual fractions of the amplitude distribution of cavitation micro and macro pulses. The superposition law is given by equation (20).
2. Erosion curve corresponding to single fraction of cavitation pulses amplitude distribution can be described using an analytical formula with at most 3 parameters defining material resistance to the loading by this fraction.
3. The energy flux delivered to the unit area of impinged surface by a monofractional cavitation cloud is proportional to the number of pulses and their amplitude square.

Following the assumptions adopted, material resistance to cavitation can be described by the **K** matrix consisting of the *U* function parameters which describe material resistance to individual fractions of cavitation loading. The matrix can be determined by minimising the expression:

$$S(\mathbf{K}) = \sum_m \sum_j \alpha_{jm} \left[\Delta V_m(t_{jm}) - A \cdot \mathrm{CEF}(\mathbf{K}, \boldsymbol{ME}_m, t_{jm}) \right]^2, \tag{24}$$

where *j* index denotes the consecutive number of a point at the erosion curve no *m,* and α_{jm} is the weigh coefficient depending on the assumed fitting criterion. In order to attain satisfactory reliability of assessment, the volume loss curves should be determined at several states of cavitation load. It is assumed that the number of these states should not be less than the number of rows in the **K** matrix. It is proposed to use an iterative approach in which the number of fractions accounted for will be increased at each major step.

ME vectors, describing individual cavitation load state, can be determined using direct and indirect methods. The direct method is based on direct measurement of pressure and force pulses using piezoelectric transducers. Due to various reasons, the technique based on piezofilm transducers [11] can be strongly recommended here. The indirect method consists in using a set of reference materials with known **K** matrix and determining the ***ME*** vector by minimising the expression

$$T(\boldsymbol{ME}) = \sum_m \sum_j \alpha_{jm} \left[\Delta V_m(t_{jm}) - A \cdot \mathrm{CEF}(\mathbf{K}_m, \boldsymbol{ME}, t_{jm}) \right]^2, \tag{25}$$

where the *m* index stands for the number of a reference material.

The ***ME*** vectors should be determined for all the conditions under which erosion tests are to be carried out. The dependence of the ***ME*** vector on operating parameters will be called dynamic cavitation characteristics of a test rig. It is clear that in case of rigs with essentially non-uniform spatial distribution of pulses only a stipulated effective ***ME*** vector can be determined. It is assumed that direct method of the ***ME*** vector determination will be used solely to determine the K matrix of reference materials. Results of the International Cavitation Erosion

Test [12] indicate that the rig most suited for such purposes is the cavitation tunnel with a slot cavitator (e.g. Erdmann-Jesnitzer test chamber). Cavitation characteristics of all other rigs will be determined using an indirect method.

4. CONCLUSION

Tests conducted under the International Cavitation Erosion Test programme [12] allow to draw some general conclusions concerning trends observed in result of laboratory investigation. It has appeared also possible to indicate rigs most suitable to form a basis for further standardisation. However, due to significant differentiation of both the design and operating parameters, quantitative correlations are confined to results of test conducted at the same rig under variable test conditions. Therefore, in addition to standardisation of selected experimental techniques, one should strive to develop methods allowing to predict material performance under variable cavitation loading conditions. A proposal of a method taking account of the effective cavitation pulses distribution has been presented in this contribution. Development of the method, its validation and implementation into experimental practice requires still substantial theoretical and experimental effort as well as acceptance by the laboratories conducting industrially oriented cavitation resistance tests.

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